Radar Pulse Compression

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Outline

- Why is pulse compression needed?
- Pulse compression, the compromise
- How it works
- Simplified view of concept
- Pulse coding
  - Phase-coded pulse
  - Chirp (linear FM)
- Receiver signal processing
- Window functions and their effects

Why is pulse compression needed?

Radar range resolution depends on the bandwidth of the received signal.

$$\rho = \frac{c\tau}{2} = \frac{c}{2B}$$

c = speed of light, ρ = range resolution,
τ = pulse duration, B = signal bandwidth

The bandwidth of a time-gated sinusoid is inversely proportional to the pulse duration.

- So short pulses are better for range resolution

Received signal strength is proportional to the pulse duration.

- So long pulses are better for signal reception

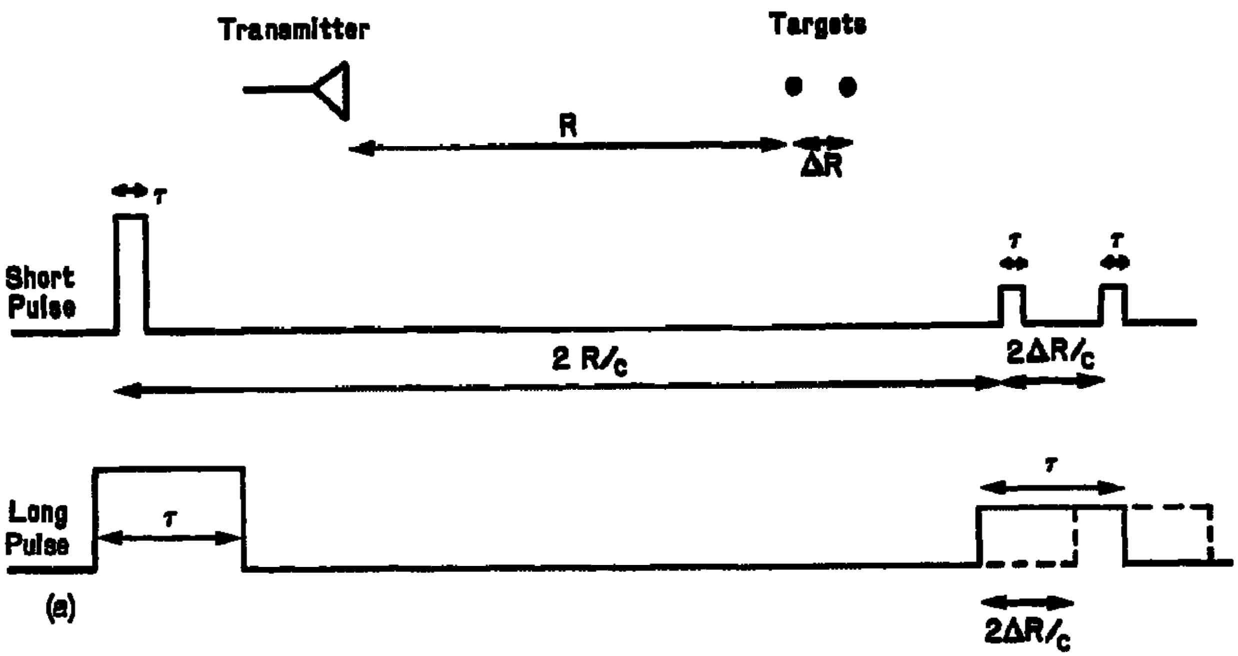
Spatial discrimination

Spatial discrimination relates to the ability to *resolve* signals from targets based on spatial position or velocity.

angle, range, velocity

Resolution is the measure of the ability to determine whether only one or more than one different targets are observed.

Range resolution, r_R, is related to signal bandwidth, B



Two targets at nearly the same range

Short pulse → higher bandwidth

Long pulse → lower bandwidth

Range resolution

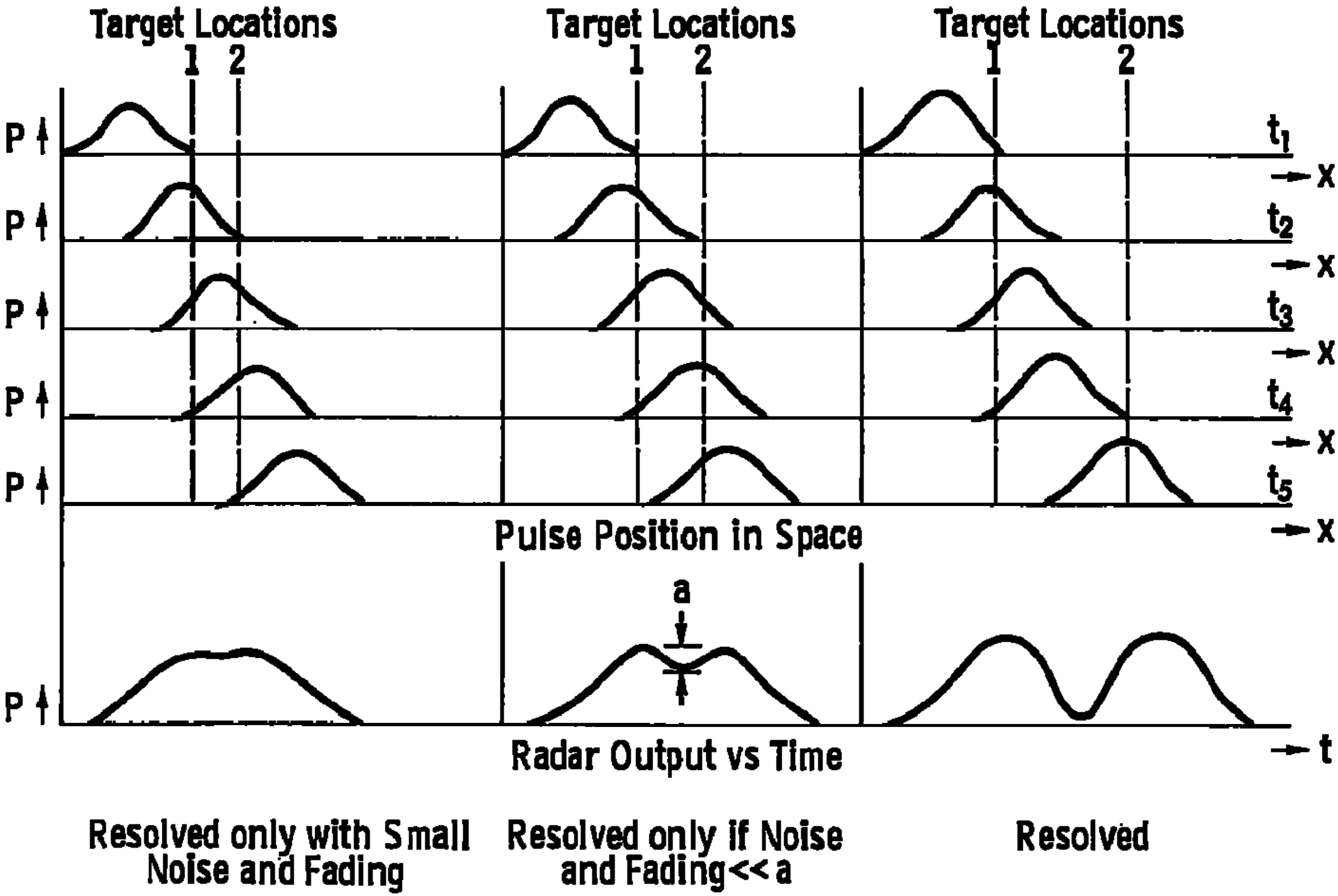


Fig. 7.19 More realistic example of range resolution.

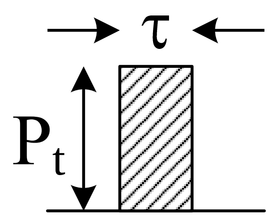
Range resolution

Clearly to obtain fine range resolution, a short pulse duration is needed.

However the amount of energy (not power) illuminating the target is a key radar performance parameter.

Energy, E, is related to the transmitted power, P_t by

$$E = \int_0^{\tau} P_t(t)\,dt$$



Therefore for a fixed transmit power, P_t, (e.g., 100 W), reducing the pulse duration, τ, reduces the energy E.

$P_t = 100$ W, $\tau = 100$ ns $\rightarrow r_R = 50$ ft, E = 10 μJ

$P_t = 100$ W, $\tau = 2$ ns $\rightarrow r_R = 1$ ft, E = 0.2 μJ

Consequently, to keep E constant, as τ is reduced, P_t must increase.

Challenges in radar

Weak received signal power (spherical spreading loss) $P_R \propto P_T / (4\pi)^2 R^4$

	Basketball court	Sear's tower	Jet aircraft	Space station	Moon
R	(94') 29 m	(1450') 442 m	(30,000') 10 km	360 km	384,400 km
$1 / (4\pi)^2 R^4$	9×10^{-9}	1.7×10^{-13}	6.3×10^{-19}	3.8×10^{-25}	2.9×10^{-37}
P_R*	0.0009 W	1.7×10^{-8} W	6.3×10^{-14} W	3.8×10^{-20} W	2.9×10^{-32} W

* assumes P_T = 100 kW = 10^5 W (KANU effective broadcast power)

Noise (anything above absolute zero radiates thermal noise) $P_N = kTB$
k = Boltzmann's constant (1.38×10^{-23} J/K)
T = temperature in Kelvin (normal room temperature is ~290 K)
B = bandwidth (Hz)
Bandwidth impacts the ability to measure range accurately or to resolve multiple targets at similar ranges, otherwise we'd set B to a very small value.

Range resolution, r_R, is bandwidth dependent, $r_R = \frac{c\tau}{2} = \frac{c}{2B}$

	10 Hz	1 kHz	200 kHz	10 MHz	300 MHz
P_N	4×10^{-20} W	4×10^{-18} W	8×10^{-16} W	4×10^{-14} W	1.2×10^{-12} W
r_R	15,000 km	150 km	750 m	15 m	50 cm

More Tx Power??

- Why not just get a transmitter that outputs more power?
- High-power transmitters present problems
  - Require high-voltage power supplies (kV)
  - Reliability problems
  - Safety issues (both from electrocution and irradiation)
  - Bigger, heavier, costlier, …

Pulse compression, the compromise

Transmit a long pulse that has a bandwidth corresponding to a short pulse

Must modulate or code the transmitted pulse

- to have sufficient bandwidth, B
- can be processed to provide the desired range resolution, ρ

Example:

Desired resolution, r_R = 15 cm (~ 6”) Required bandwidth, B = 1 GHz (10^9 Hz)

Required pulse energy, E = 1 mJ $E(J) = P(W) \cdot \tau(s)$

Brute force approach

Raw pulse duration, τ = 1 ns (10^{-9} s) Required transmitter power, P = **1 MW !**

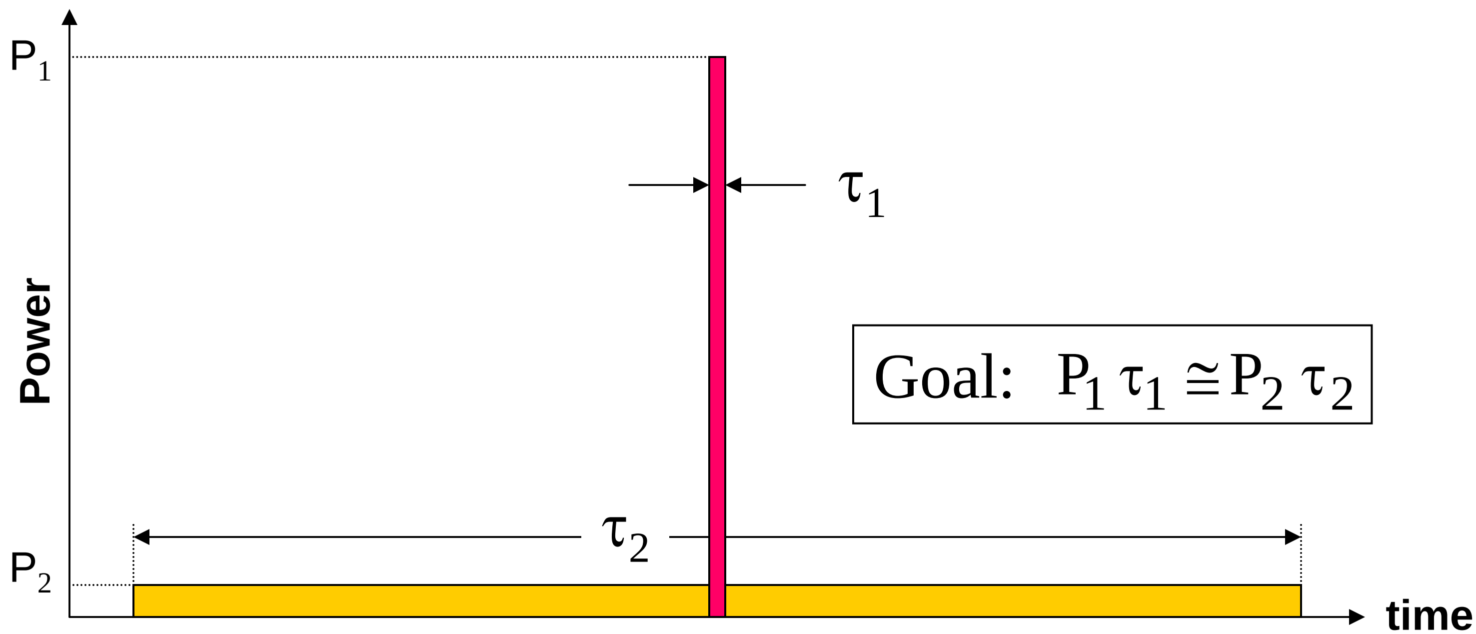
Pulse compression approach

Pulse duration, τ = 0.1 ms (10^{-4} s) Required transmitter power, P = **100 W**

Simplified view of concept

Energy content of long-duration, low-power pulse will be comparable to that of the short-duration, high-power pulse

$$\tau_1 \ll \tau_2 \text{ and } P_1 \gg P_2$$



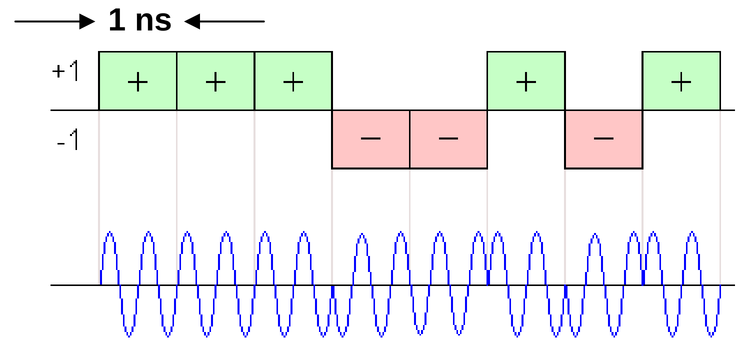
Pulse coding

Long duration pulse is coded to have desired bandwidth.

Various ways to code pulse.

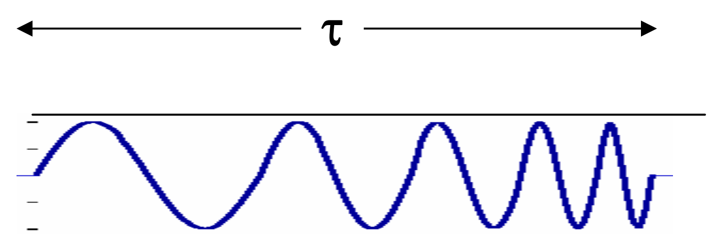
Phase code short segments

Each segment duration = 1 ns



Linear frequency modulation (chirp)

$$s(t) = A\cos\left(2\pi\left(f_C t + 0.5\,k\,t^2\right) + \phi_C\right)$$



for $0 \le t \le \tau$

f_C is the starting frequency (Hz)

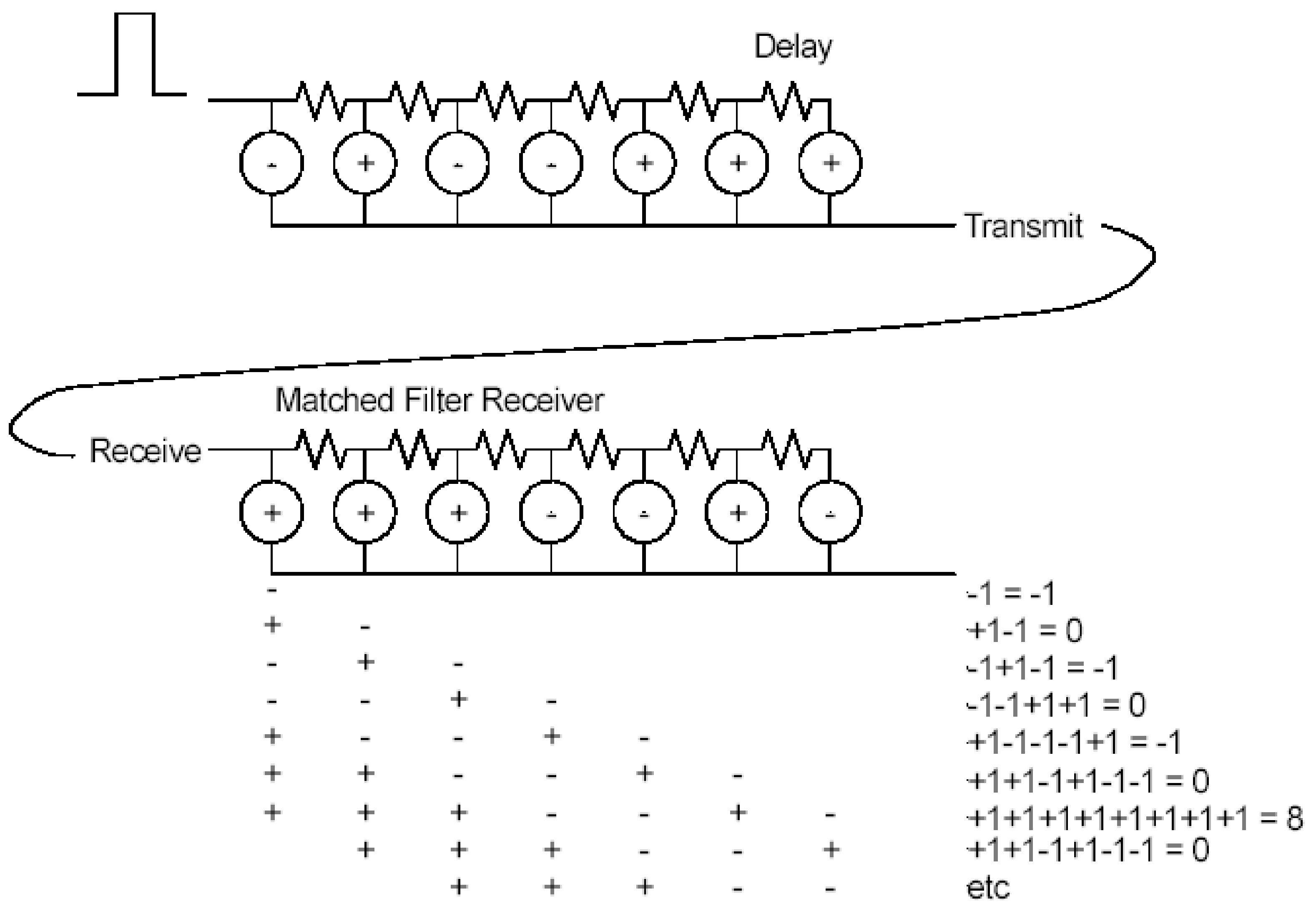
k is the chirp rate (Hz/s)

$B = k\tau = 1$ GHz

Choice driven largely by required complexity of receiver electronics

Receiver signal processing

phase-coded pulse generation and compression

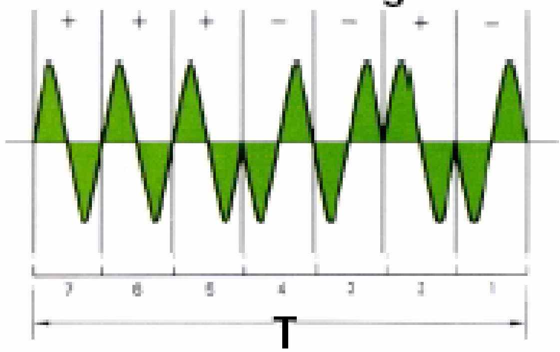


Receiver signal processing

phase-coded pulse compression

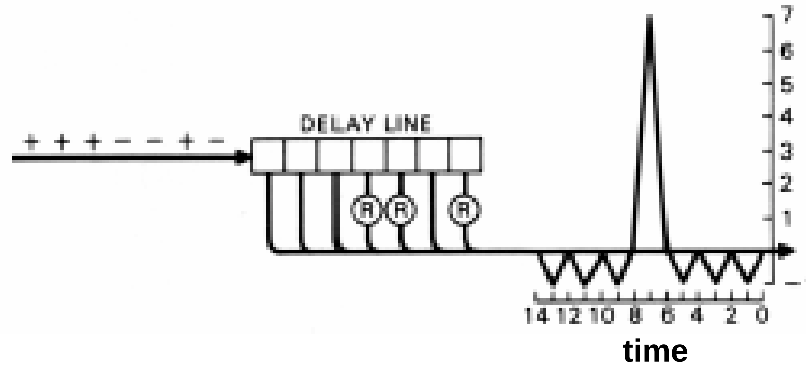
BINARY-PHASE CODED PULSE:

- Barker Code of Length N = 7



BINARY-PHASE DECODER FOR PULSE COMP. :

- For Barker Codes, PSL = -20log(N)

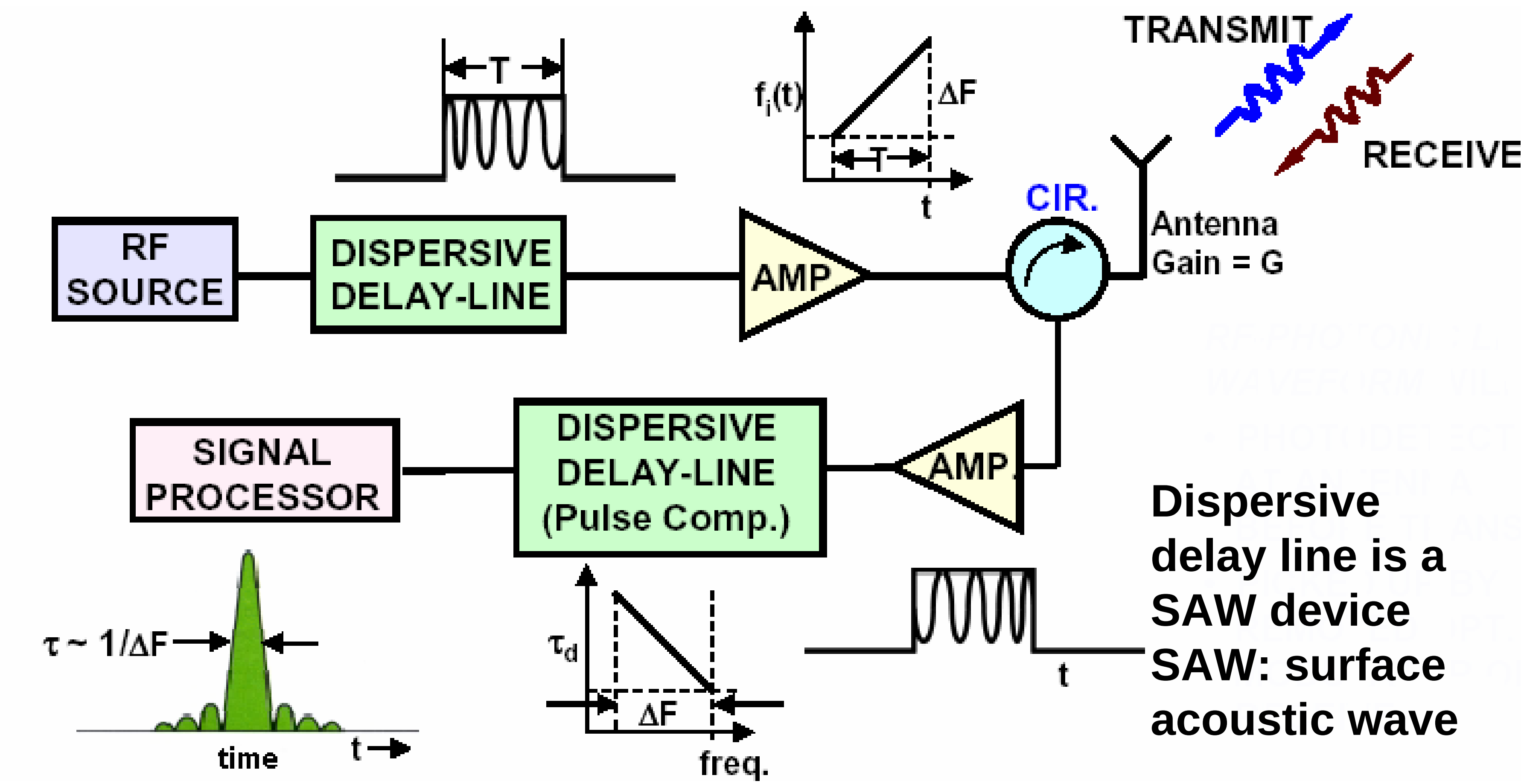


Correlation process may be performed in analog or digital domain. A disadvantage of this approach is that the data acquisition system (A/D converter) must operate at the full system bandwidth (e.g., 1 GHz in our example).

PSL: peak sidelobe level (refers to time sidelobes)

Receiver signal processing

chirp generation and compression



Dispersive delay line is a SAW device

SAW: surface acoustic wave

Chirp radar

Transmit a long-duration, FM pulse.

Correlate the received signal with a linear FM waveform to produce range dependent target frequencies.

Signal processing (pulse compression) converts frequency into range.

Key parameters:

B, chirp bandwidth

τ, Tx pulse duration

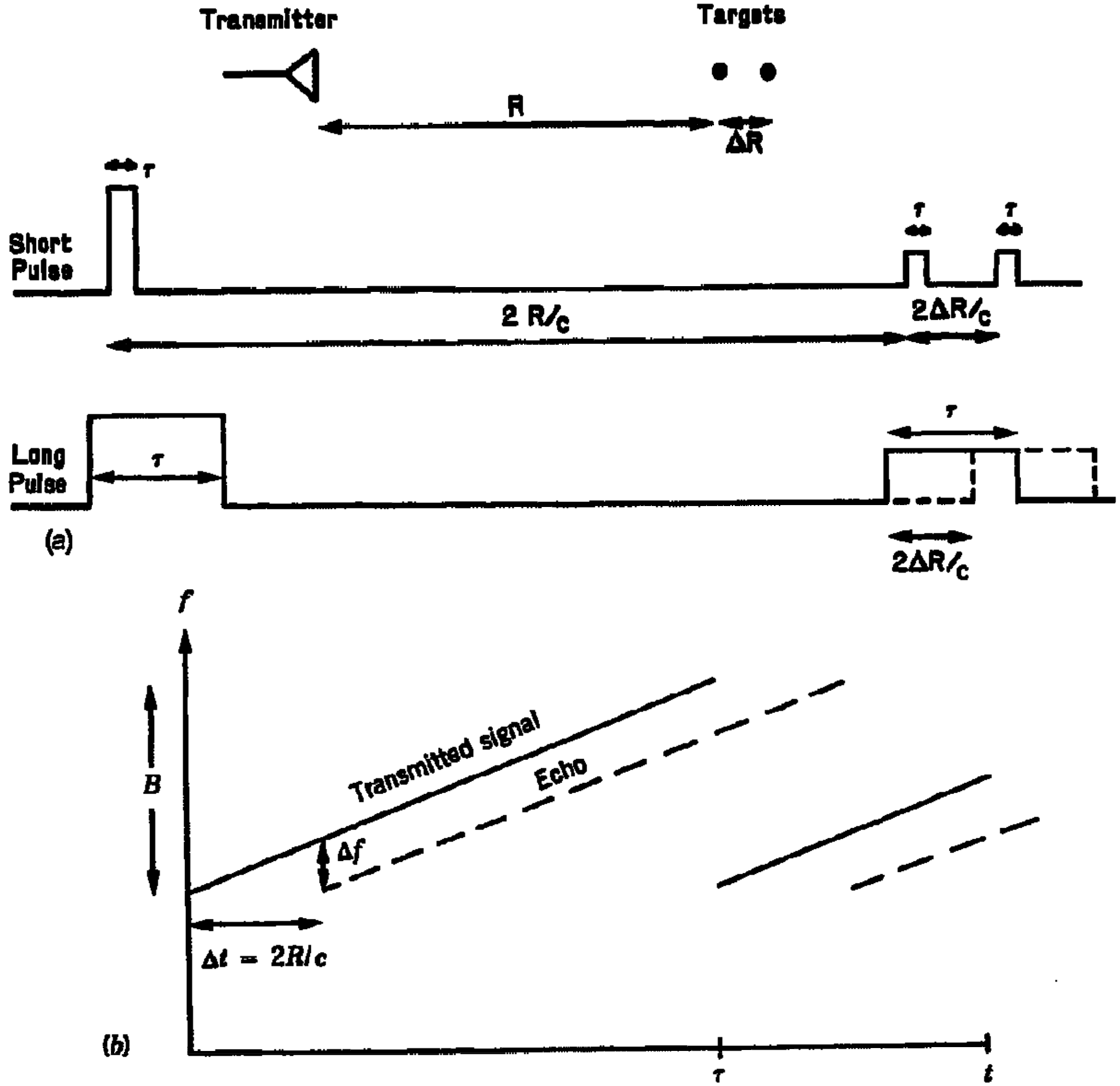
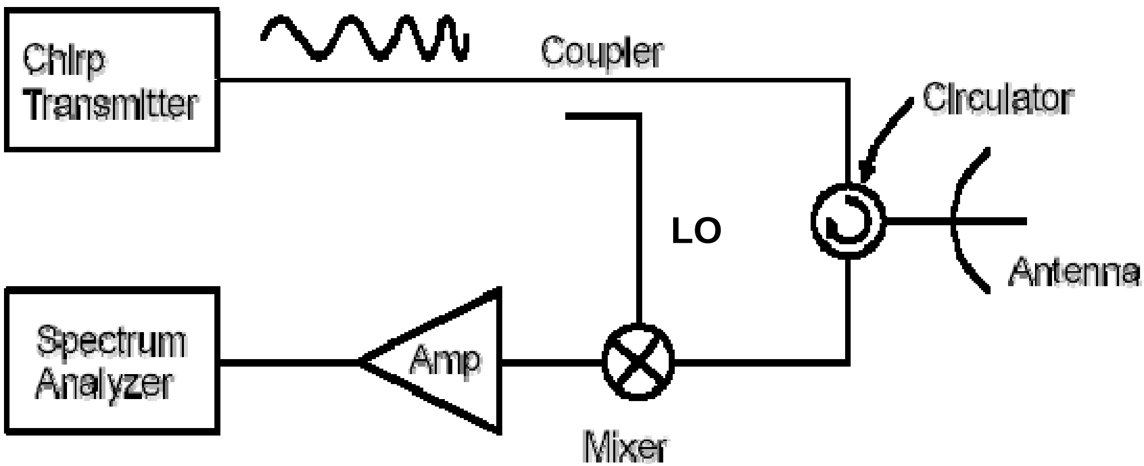
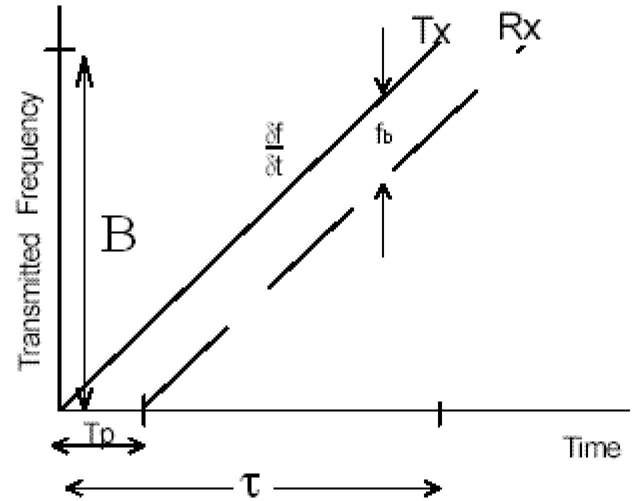
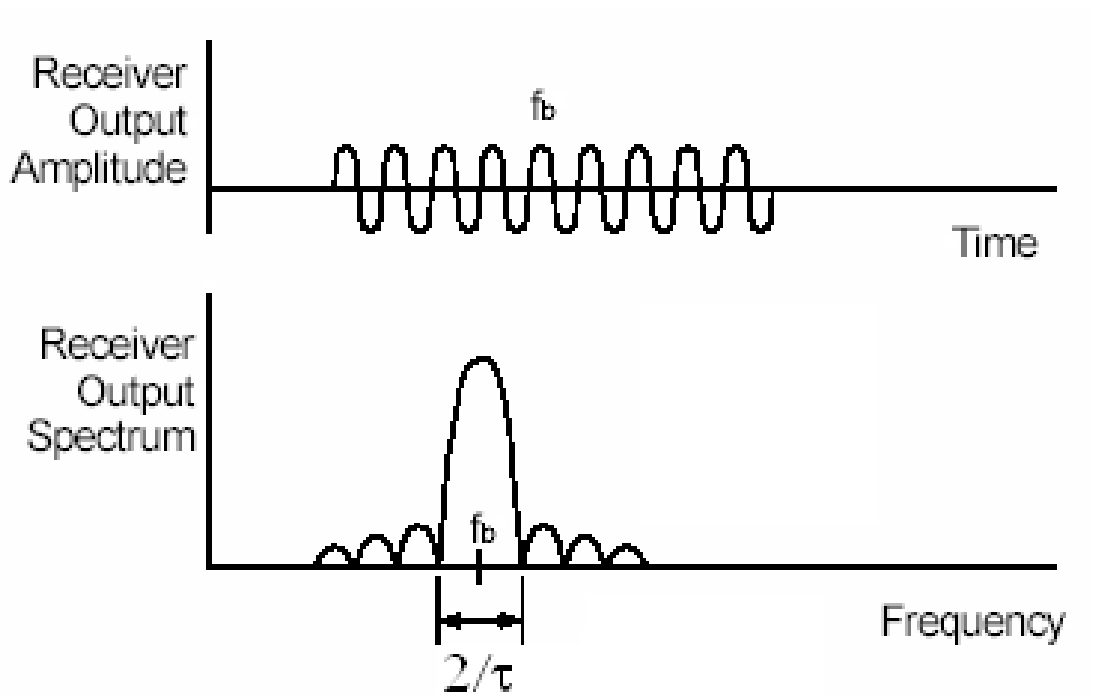


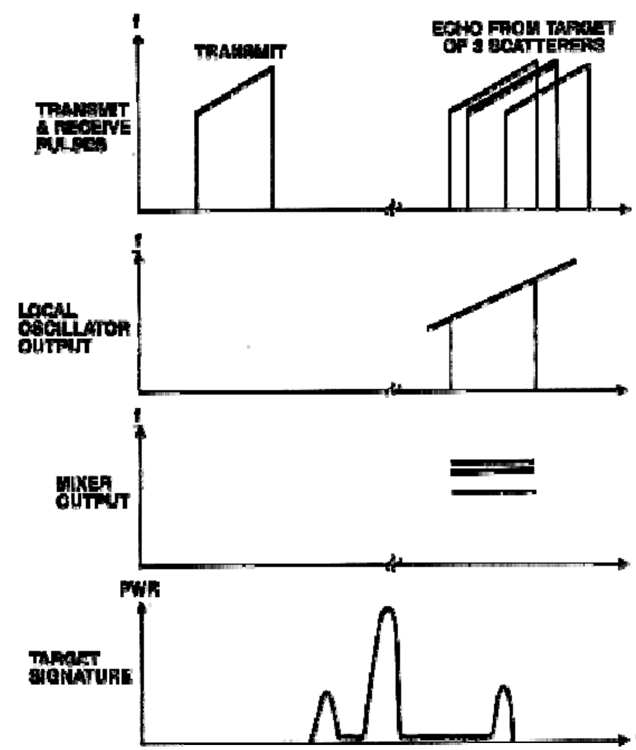
Figure 6-22. (a) The range measurement technique using pulsed radar. Separation of targets would require a short pulse such that $\tau < 2\,\Delta R/c$. (b) The range measurement technique using FM sweep radar.

Stretch chirp processing

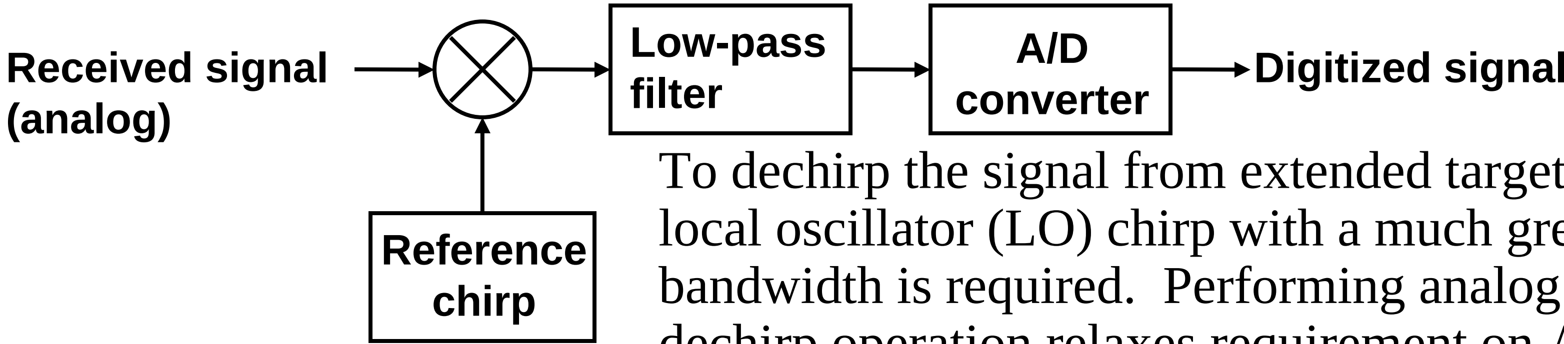






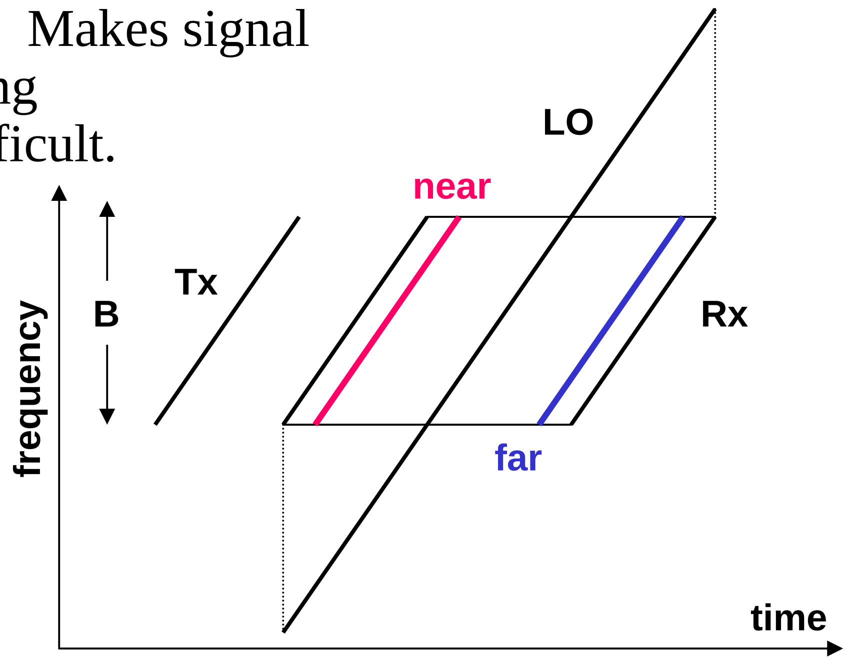


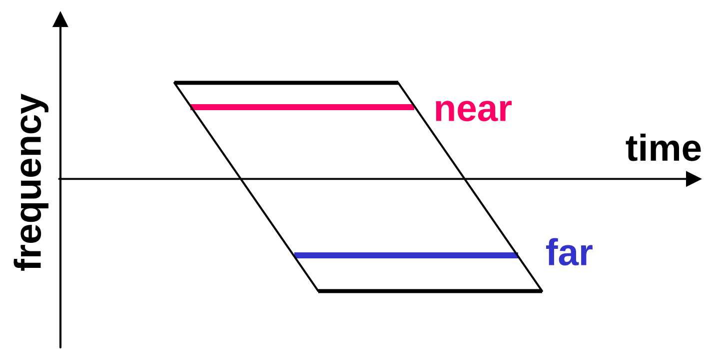
Challenges with stretch processing



To dechirp the signal from extended targets, a local oscillator (LO) chirp with a much greater bandwidth is required. Performing analog dechirp operation relaxes requirement on A/D converter.

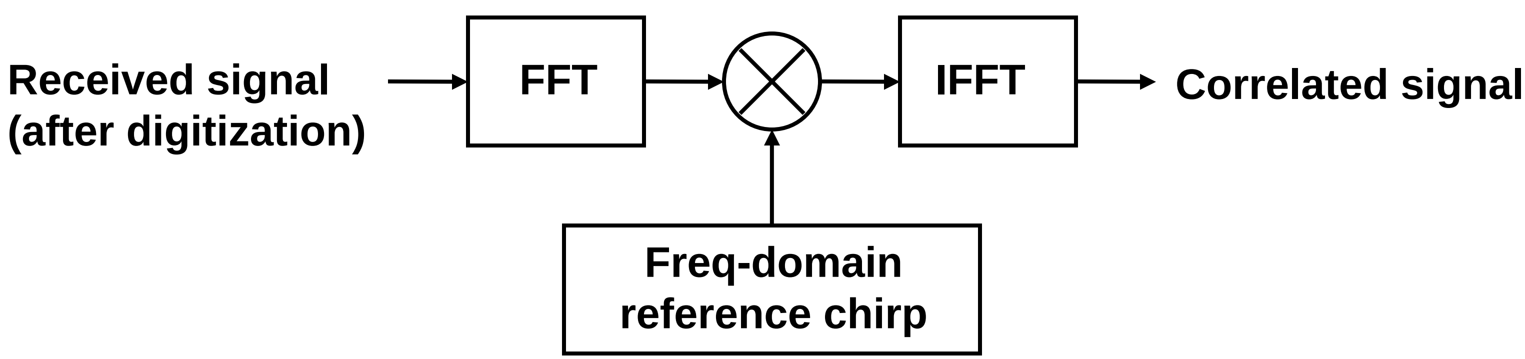
Echos from targets at various ranges have different start times with constant pulse duration. Makes signal processing
more difficult.



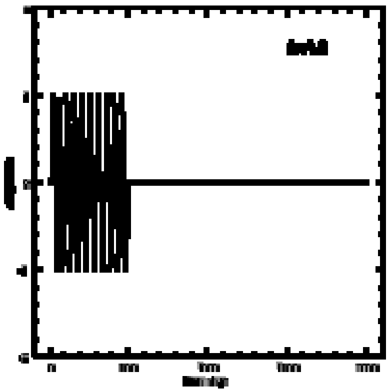


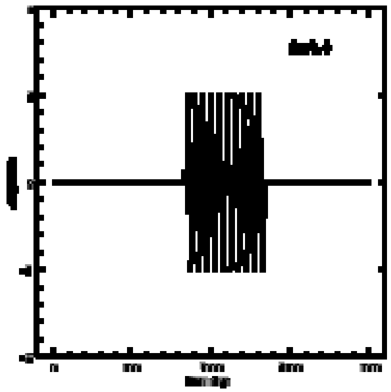
Correlation processing of chirp signals

- Avoids problems associated with stretch processing
- Takes advantage of fact that convolution in time domain equivalent to multiplication in frequency domain
  - Convert received signal to freq domain (FFT)
  - Multiply with freq domain version of reference chirp function
  - Convert product back to time domain (IFFT)



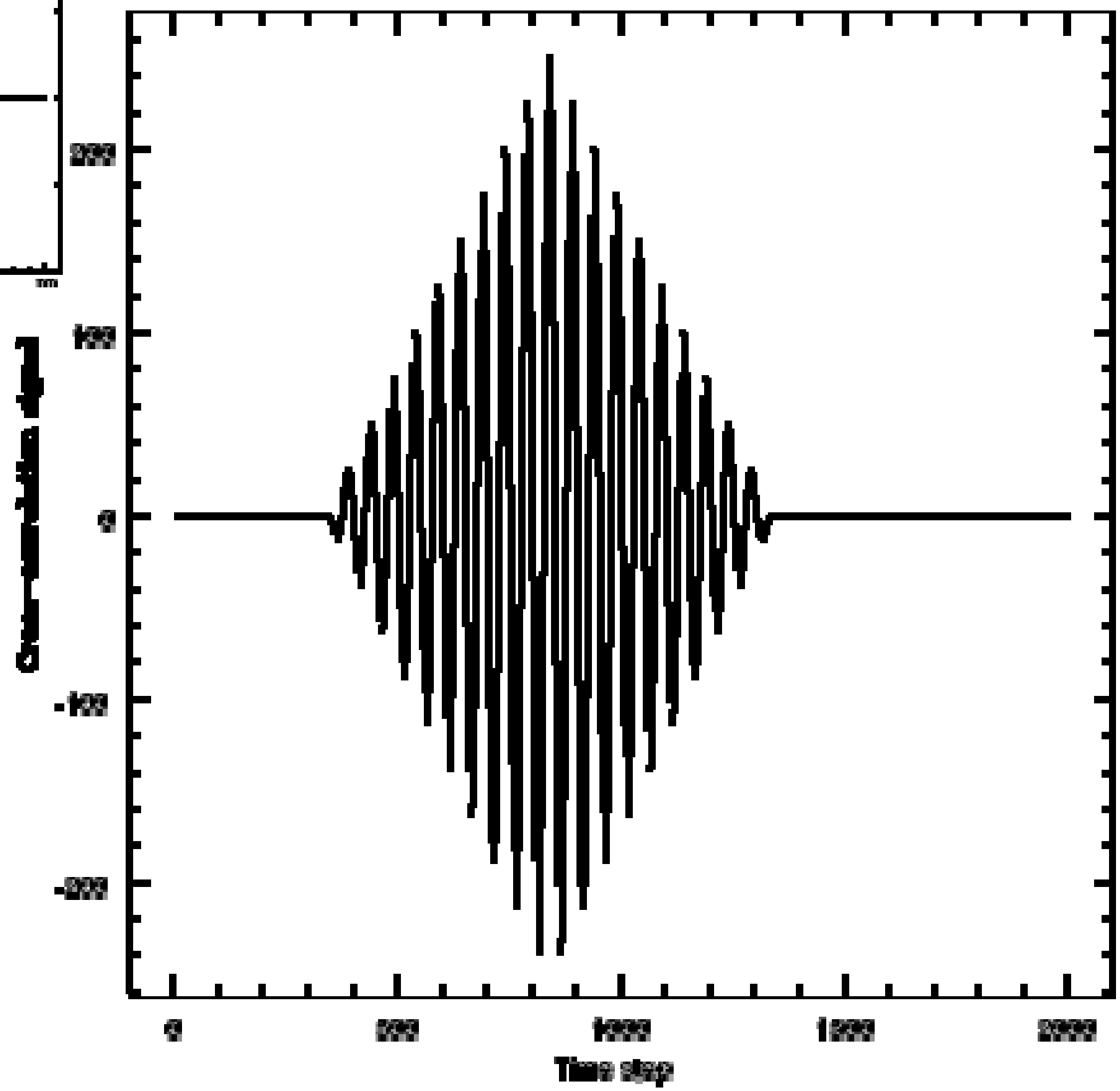
Signal correlation examples



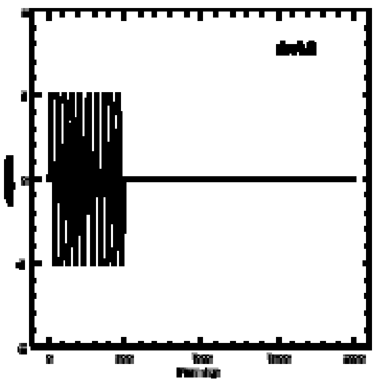


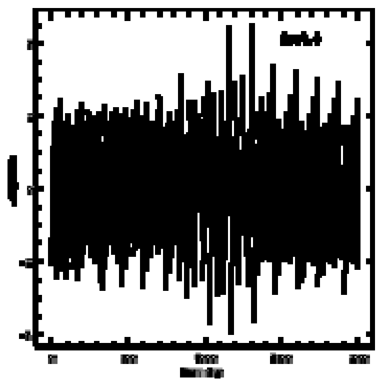
<u>Input waveform #1</u>
High-SNR gated sinusoid, no delay

<u>Input waveform #2</u>
High-SNR gated sinusoid, ~800 count delay



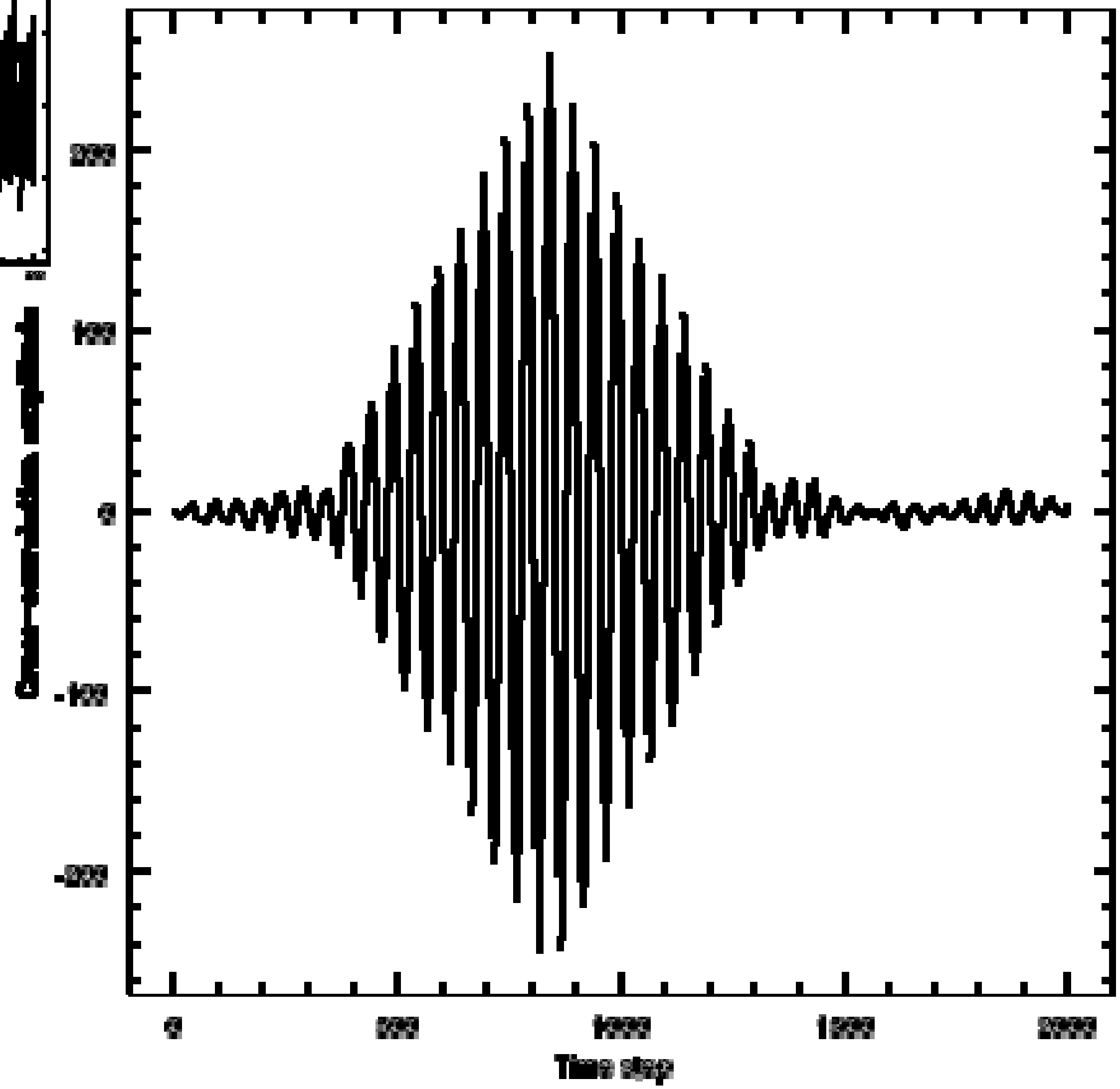
Signal correlation examples





<u>**Input waveform #1**</u>
High-SNR gated sinusoid, no delay

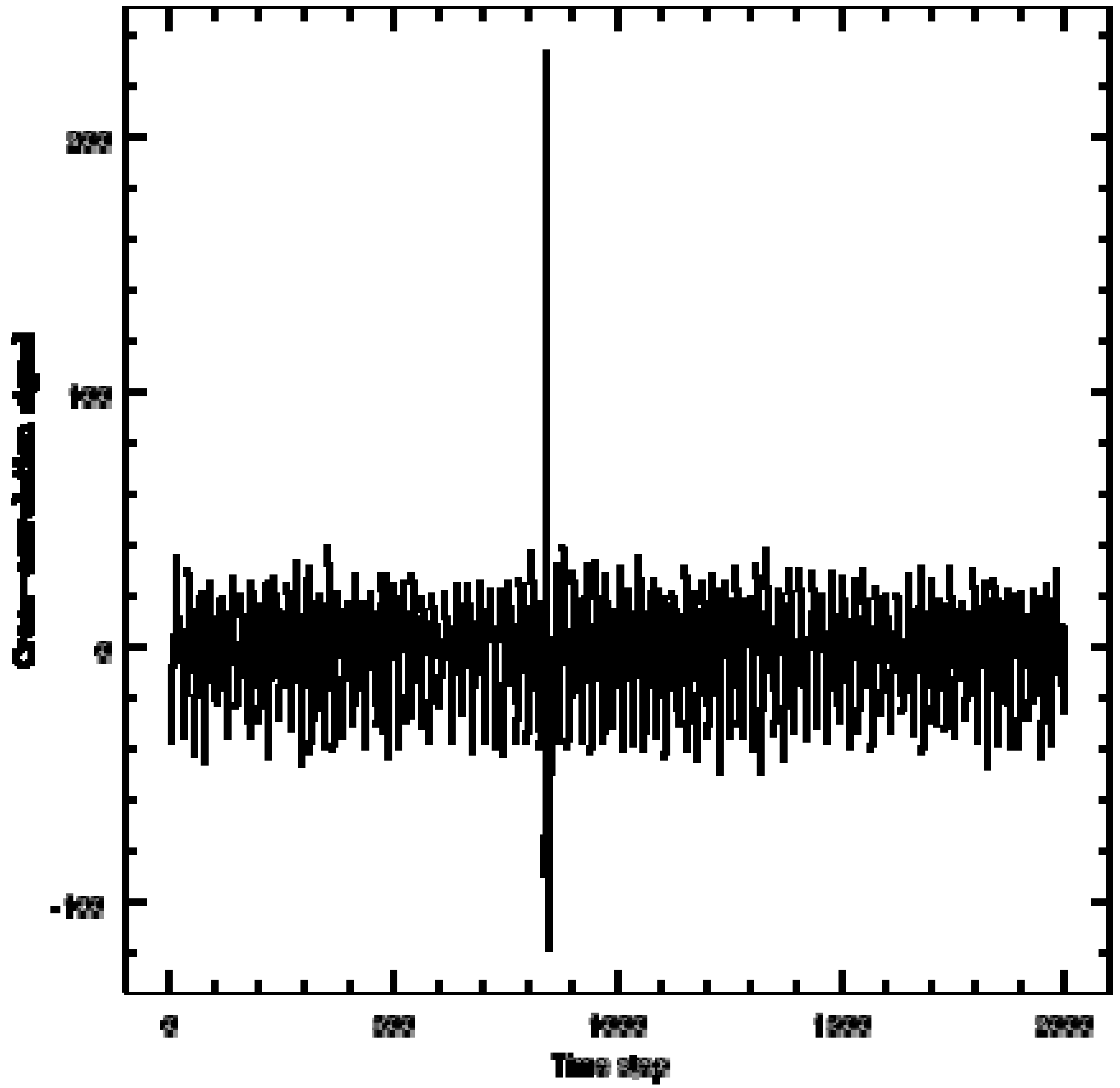
<u>**Input waveform #2**</u>
Low-SNR gated sinusoid, ~800 count delay



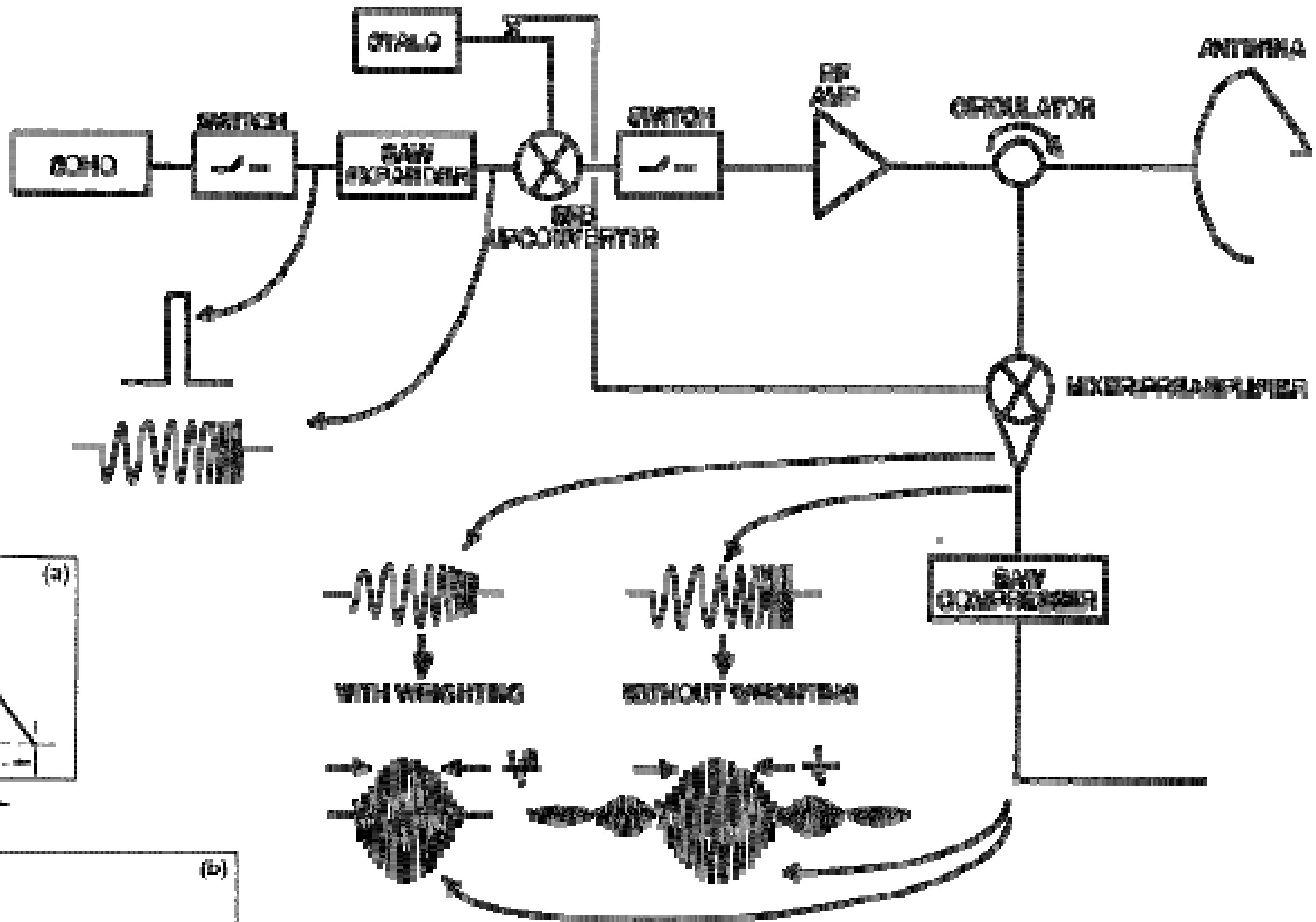
Signal correlation examples

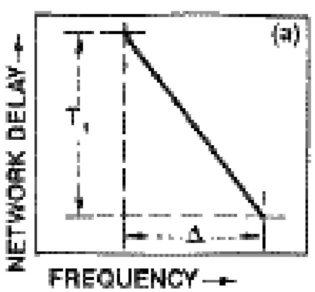
<u>Input waveform #1</u>
High-SNR gated chirp, no delay

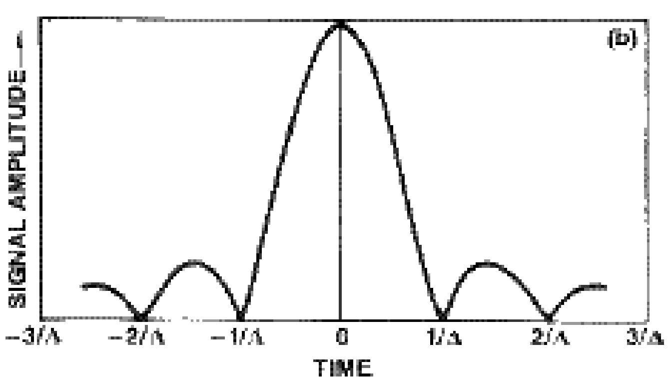
<u>Input waveform #2</u>
Low-SNR gated chirp, ~800 count delay



Chirp pulse compression and sidelobes







Peak sidelobe level can be controlled by introducing a weighting function -- however this has side effects.

Window functions and their effects

Weighting Function	Peak Sidelobe Level	S/N Loss	Relative Mainlobe Width
Uniform	-13.2	0	1
$0.33+0.66\cos^2(\pi f/\beta)$	-25.7	0.55	1.23
$\cos^2(\pi f/\beta)$	-31.7	1.76	1.65
Taylor (n=8)	-40	1.14	1.41
Dolph Chebyshev	-40	-	1.35
Hamming	-42.8	1.34	1.5

Time sidelobes are an side effect of pulse compression.

Windowing the signal prior to frequency analysis helps reduce the effect.

Some common weighting functions and key characteristics

Less common window functions used in radar applications and their key characteristics

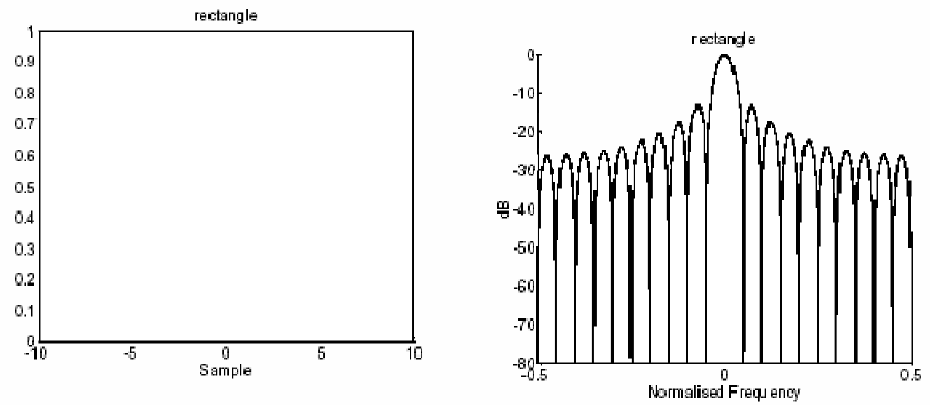
Window	Rect angle	Ham ming	Black man	Blackman-Harris			
				X3	M3	X4	M4
Worst Sidelobe (dB)	-13	-43	-58	-61	-67	-74	-92
3dB beamwidth (bins)	0.89	1.3	1.68	1.56	1.66	1.74	1.9
Resolution (bin)	1.21	1.81	2.35	2.19	2.2	2.44	2.72
Scalloping Loss (dB)	3.92	1.78	1.1	1.27	1.13	1.03	0.83
SNR Loss (dB)	0	1.34	2.37	2.07	2.33	2.53	3.02
Main Lobe Width (bins)	2	4	6	6	6	8	8
a0	1	0.54	0.42	0.44959	0.42323	0.40217	0.35875
a1		0.46	0.50	0.44364	0.49755	0.49703	0.48829
a2			0.08	0.05677	0.07922	0.09392	0.14128
a3						0.00183	0.01168
$W(n)=a0-a1\cos[2\pi(n-1)/N]+a2\cos[4\pi(n-1)/N]-a3\cos[6\pi(n-1)/N]$							

Window functions

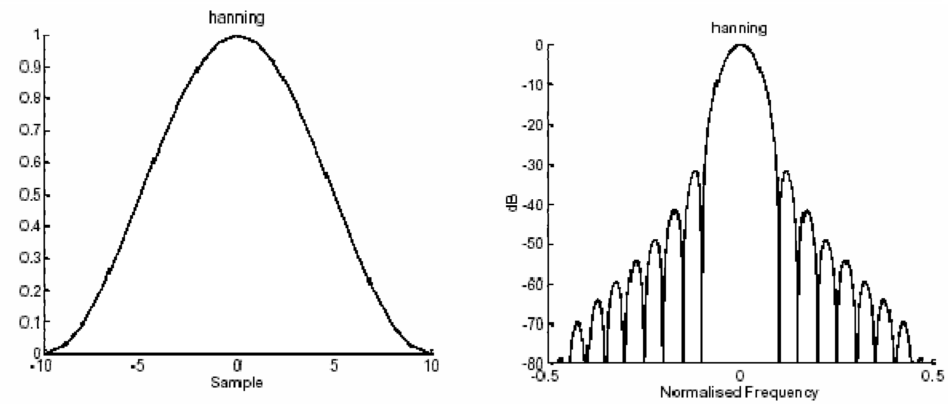
Basic function: $c_k = \cos\left(2k\pi(n - \tfrac{1}{2}N)/N\right)$

a and b are the –6-dB and -∞ normalized bandwidths

Rectangular: $w(n) \equiv 1$
a=1.21, b=2
Sidelobe = -13dB

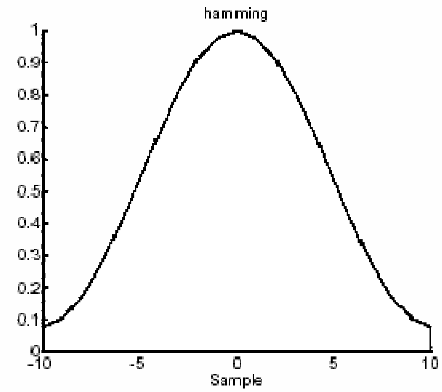


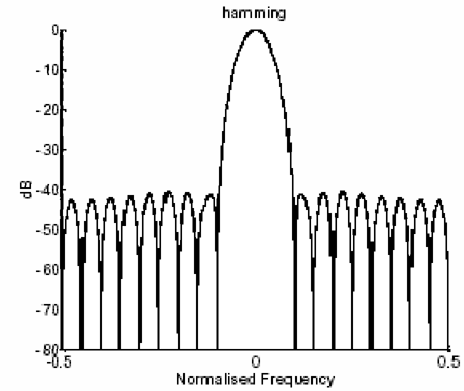
Hanning: $0.5 + 0.5c_1$
a=1.65, b=4
Sidelobe = –23dB



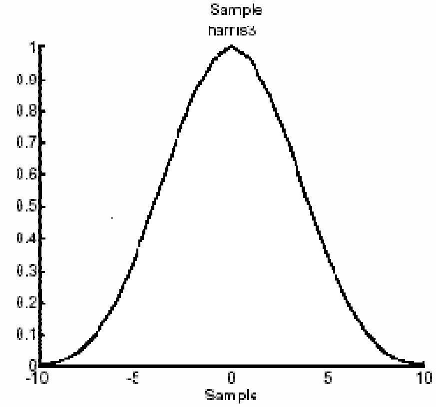
Window functions

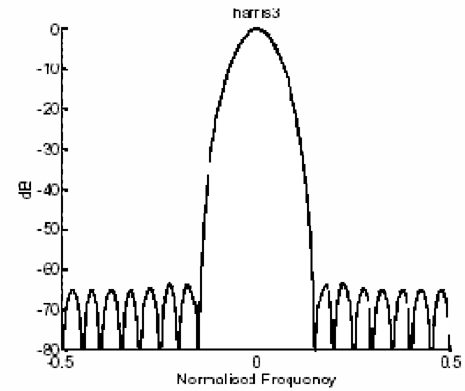
Hamming: $0.54 + 0.46c_1$
a=1.81, b=4
Sidelobe = –43dB



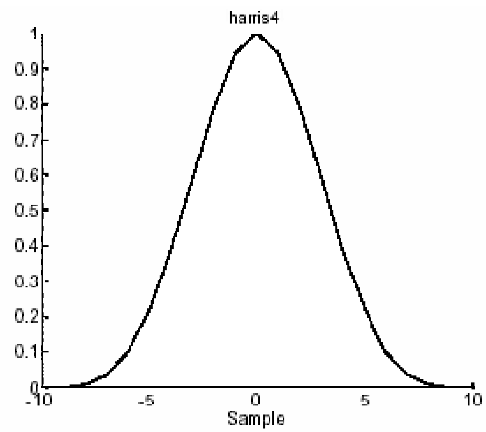


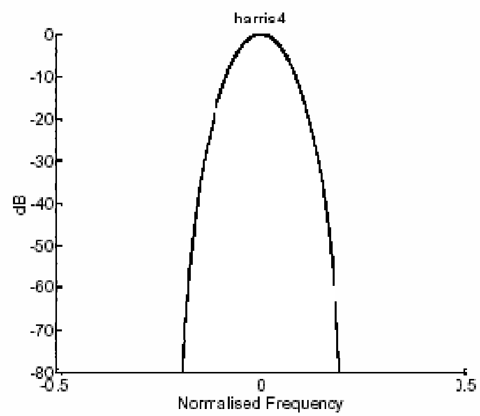
Blackman-Harris: 3 term
$0.423 + 0.498c_1 + 0.079c_2$
a=1.81, b=6
Sidelobe = –67dB





Blackman-Harris: 4 term
$0.359 + 0.488c_1 + 0.141c_2 + 0.012c_3$
a=2.72, b=8
Sidelobe = –92dB





Detailed example of chirp pulse compression

received signal

$$s(t) = a\cos\left(2\pi\left(f_C t + 0.5\,k\,t^2\right) + \phi_C\right)$$

dechirp analysis

$$s(t)\,s(t-\tau) = a\cos\left(2\pi\left(f_C t + 0.5\,k\,t^2\right) + \phi_C\right) a\cos\left(2\pi\left(f_C (t-\tau) + 0.5\,k\,(t-\tau)^2\right) + \phi_C\right)$$

which simplifies to

$$s(t)\,s(t-\tau) = \frac{a^2}{2}\left[\begin{array}{c}\cos\left(2\pi f_C \tau + 2\pi k\,t\,\tau - \pi k \tau^2\right) \\ + \cos\left(2\pi\left(k\,t^2 + 2 f_C t - k\,\tau\,t + 0.5\,k\,\tau^2 - f_C \tau\right) + 2\phi_C\right)\end{array}\right]$$

sinusoidal term

chirp-squared term

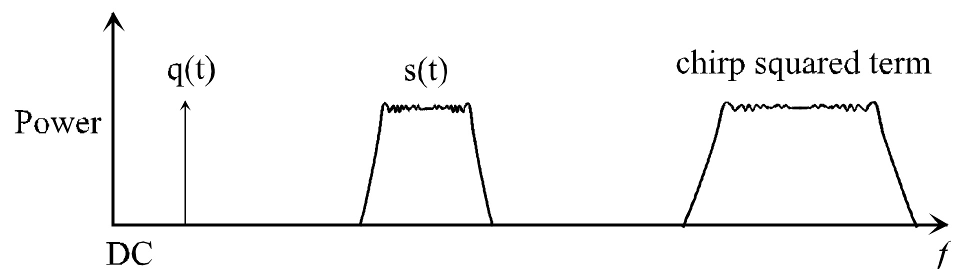
quadratic frequency dependence

linear frequency dependence

phase terms

after lowpass filtering to reject harmonics

$$q(t) = \frac{a^2}{2}\cos\left(2\pi\left(f_C \tau + k\,\tau\,t - 0.5\,k\,\tau^2\right)\right)$$



Pulse compression effects on SNR and blind range

SNR improvement due to pulse compression: $B\tau$

$$SNR_{compress} = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T B F} B\tau$$

Case 1: P_t = 1 MW, τ = 1 ns, B = 1 GHz, E = 1 mJ

For a given R, G_t, G_r, λ, σ: SNR_{video} = 10 dB

$B\tau$ = 1 or 0 dB

$SNR_{compress} = SNR_{video}$ = 10 dB

Blind range = $c\tau/2$ = **0.15 m**

Case 2: P_t = 100 W, τ = 0.1 ms, B = 1 GHz, E = 1 mJ

For a same R, G_t, G_r, λ, σ: SNR_{video} = -30 dB

$B\tau$ = 100,000 or 50 dB

$SNR_{compress}$ = 10 dB

Blind range = $c\tau/2$ = **15 km**

Conclusions

Pulse compression allows us to use a reduced transmitter power and still achieve the desired range resolution.

The costs of applying pulse compression include:

– added transmitter and receiver complexity
– must contend with time sidelobes
– increased blind range

The advantages generally outweigh the disadvantages so pulse compression is used widely.