

# Radar Pulse Compression

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# Outline

- Why is pulse compression needed?
- Pulse compression, the compromise
- How it works
- Simplified view of concept
- Pulse coding
  - Phase-coded pulse
  - Chirp (linear FM)
- Receiver signal processing
- Window functions and their effects

# Why is pulse compression needed?

Radar range resolution depends on the bandwidth of the received signal.

$$\rho = \frac{c\tau}{2} = \frac{c}{2B}$$

$c$  = speed of light,  $\rho$  = range resolution,  
 $\tau$  = pulse duration,  $B$  = signal bandwidth

The bandwidth of a time-gated sinusoid is inversely proportional to the pulse duration.

- So short pulses are better for range resolution

Received signal strength is proportional to the pulse duration.

- So long pulses are better for signal reception

# More Tx Power??

Why not just get a transmitter that outputs more power?

High-power transmitters present problems

- Require high-voltage power supplies (kV)

- Reliability problems

- Safety issues (both from electrocution and irradiation)

- Bigger, heavier, costlier, ...

# Pulse compression, the compromise

Transmit a long pulse that has a bandwidth corresponding to a short pulse

Must modulate or code the transmitted pulse

- to have sufficient bandwidth,  $B$
- can be processed to provide the desired range resolution,  $\rho$

## Example:

Desired resolution,  $\rho = 15 \text{ cm}$  ( $\sim 6''$ )

Required bandwidth,  $B = 1 \text{ GHz}$  ( $10^9 \text{ Hz}$ )

Required pulse energy,  $E = 1 \text{ mJ}$

$E(\text{J}) = P(\text{W}) \cdot \tau(\text{s})$

## **Brute force approach**

Raw pulse duration,  $\tau = 1 \text{ ns}$  ( $10^{-9} \text{ s}$ )

Required transmitter power,  $P = \mathbf{1 \text{ MW} !}$

## **Pulse compression approach**

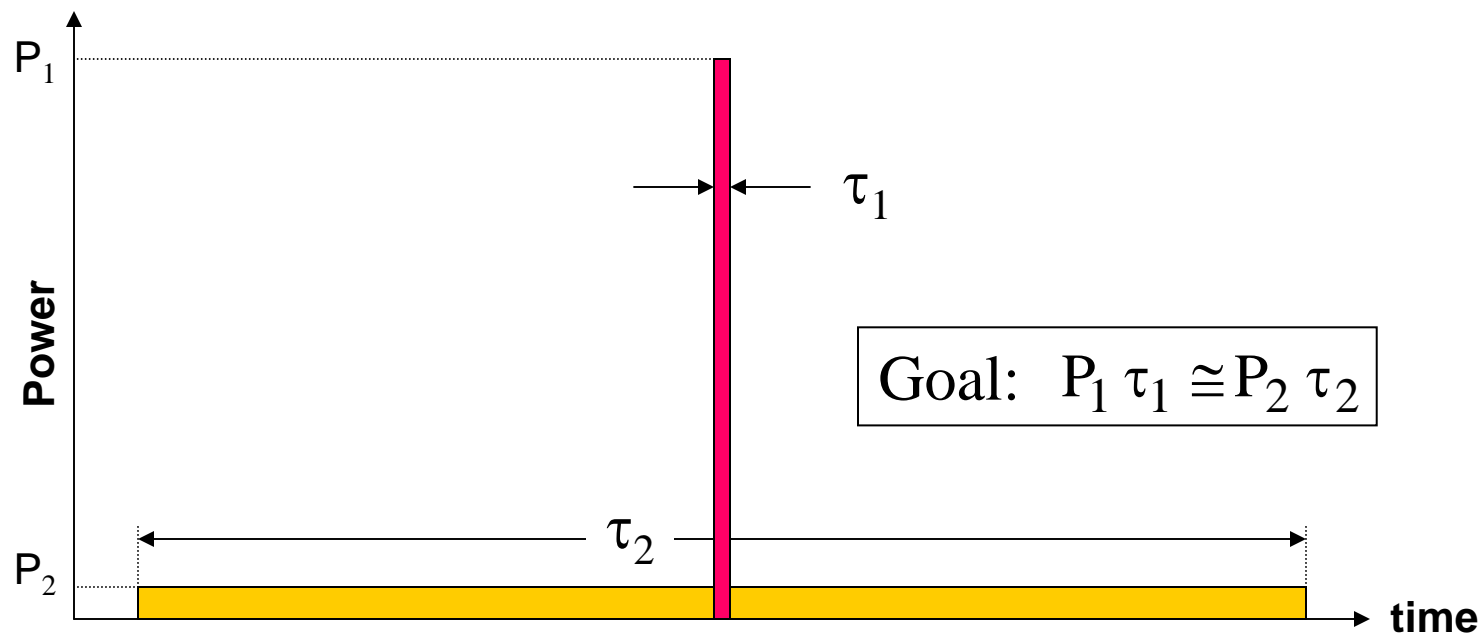
Pulse duration,  $\tau = 0.1 \text{ ms}$  ( $10^{-4} \text{ s}$ )

Required transmitter power,  $P = \mathbf{100 \text{ W}}$

# Simplified view of concept

Energy content of long-duration, low-power pulse  
will be comparable to that of the short-duration,  
high-power pulse

$$\tau_1 \ll \tau_2 \text{ and } P_1 \gg P_2$$



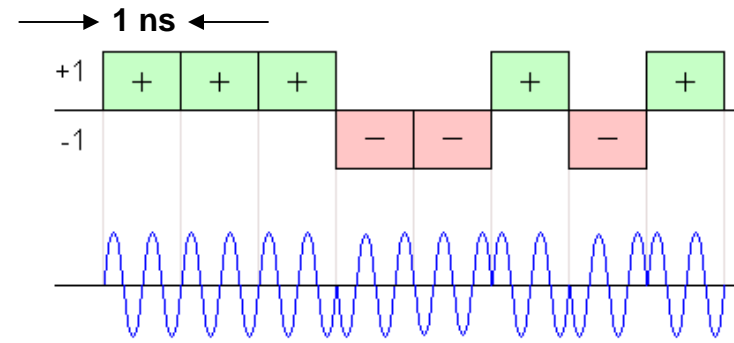
# Pulse coding

Long duration pulse is coded to have desired bandwidth.

Various ways to code pulse.

Phase code short segments

Each segment duration = 1 ns



Linear frequency modulation (chirp)

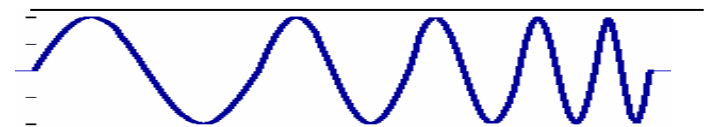
$$s(t) = A \cos \left( 2 \pi \left( f_c t + 0.5 k t^2 \right) + \phi_c \right)$$

for  $0 \leq t \leq \tau$

$f_c$  is the starting frequency (Hz)

$k$  is the chirp rate (Hz/s)

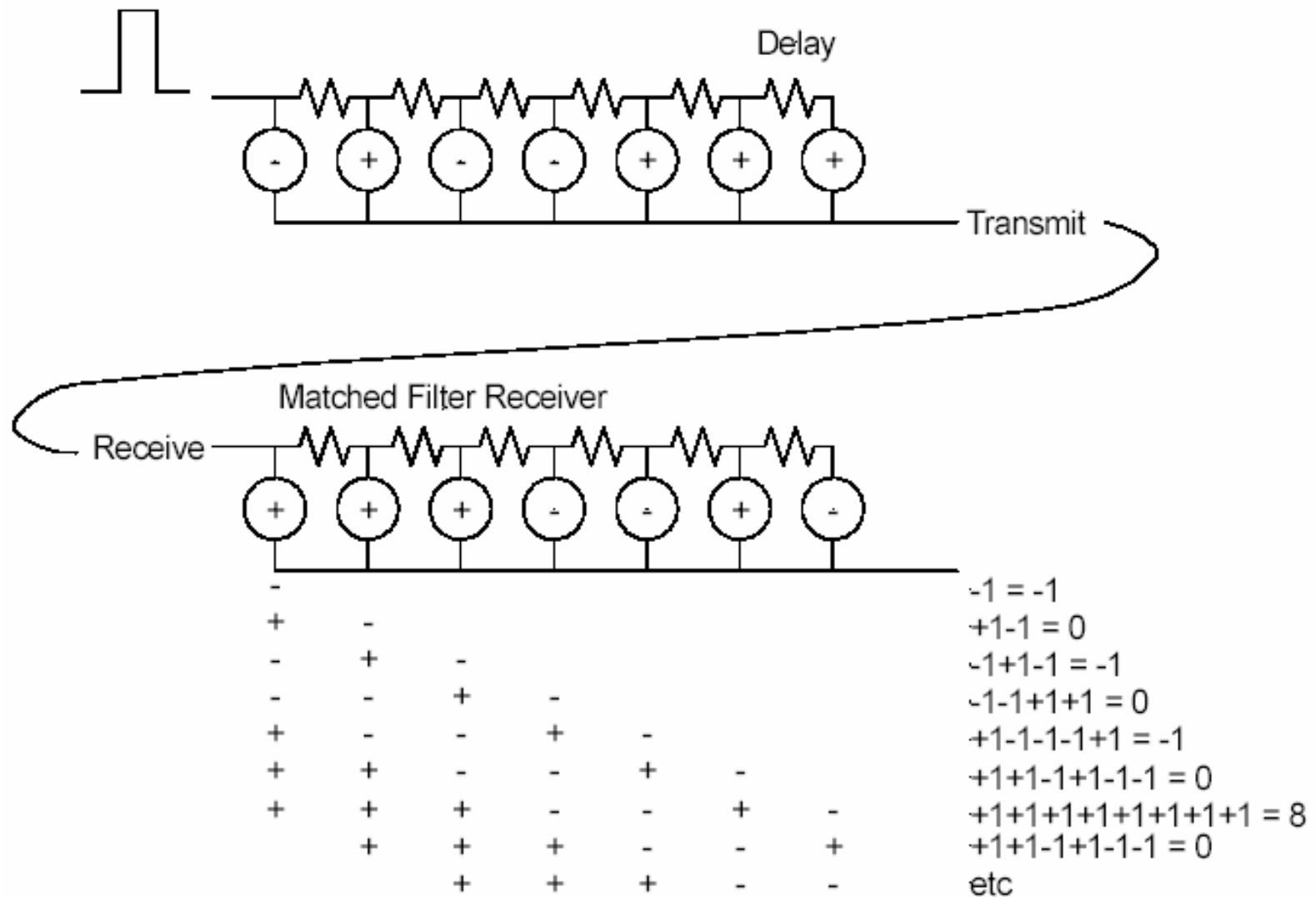
$B = k\tau = 1 \text{ GHz}$



Choice driven largely by required complexity of receiver electronics

# Receiver signal processing

## phase-coded pulse generation and compression



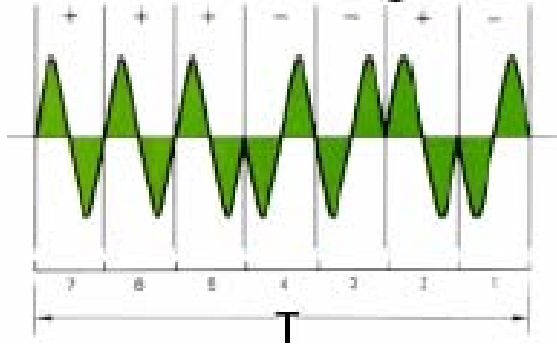


# Receiver signal processing

## phase-coded pulse compression

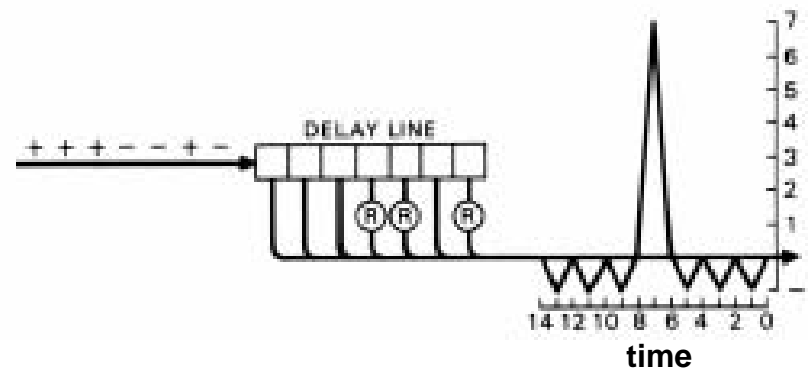
### BINARY-PHASE CODED PULSE:

- Barker Code of Length  $N = 7$



### BINARY-PHASE DECODER FOR PULSE COMP. :

- For Barker Codes,  $PSL = -20\log(N)$

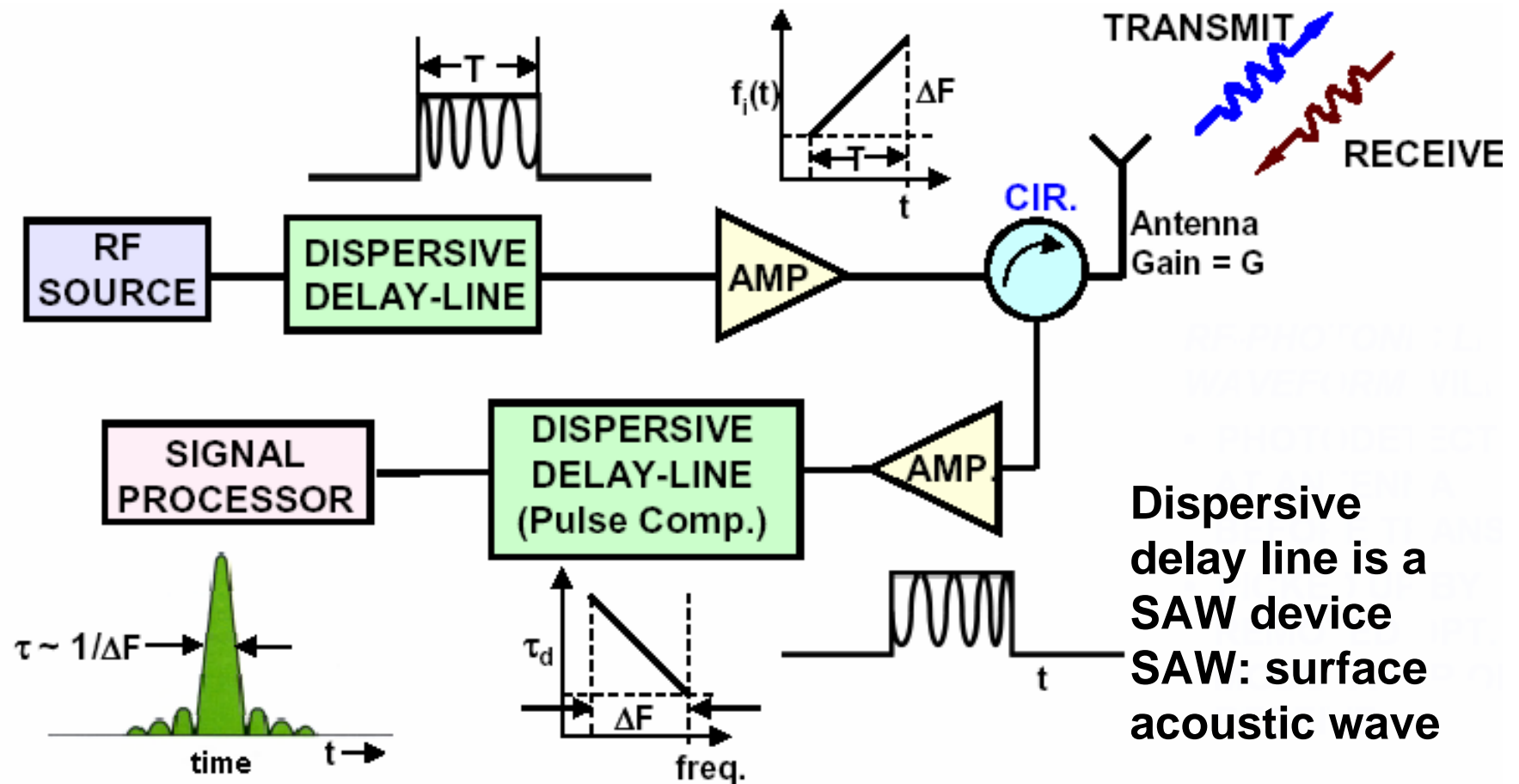


Correlation process may be performed in analog or digital domain. A disadvantage of this approach is that the data acquisition system (A/D converter) must operate at the full system bandwidth (e.g., 1 GHz in our example).

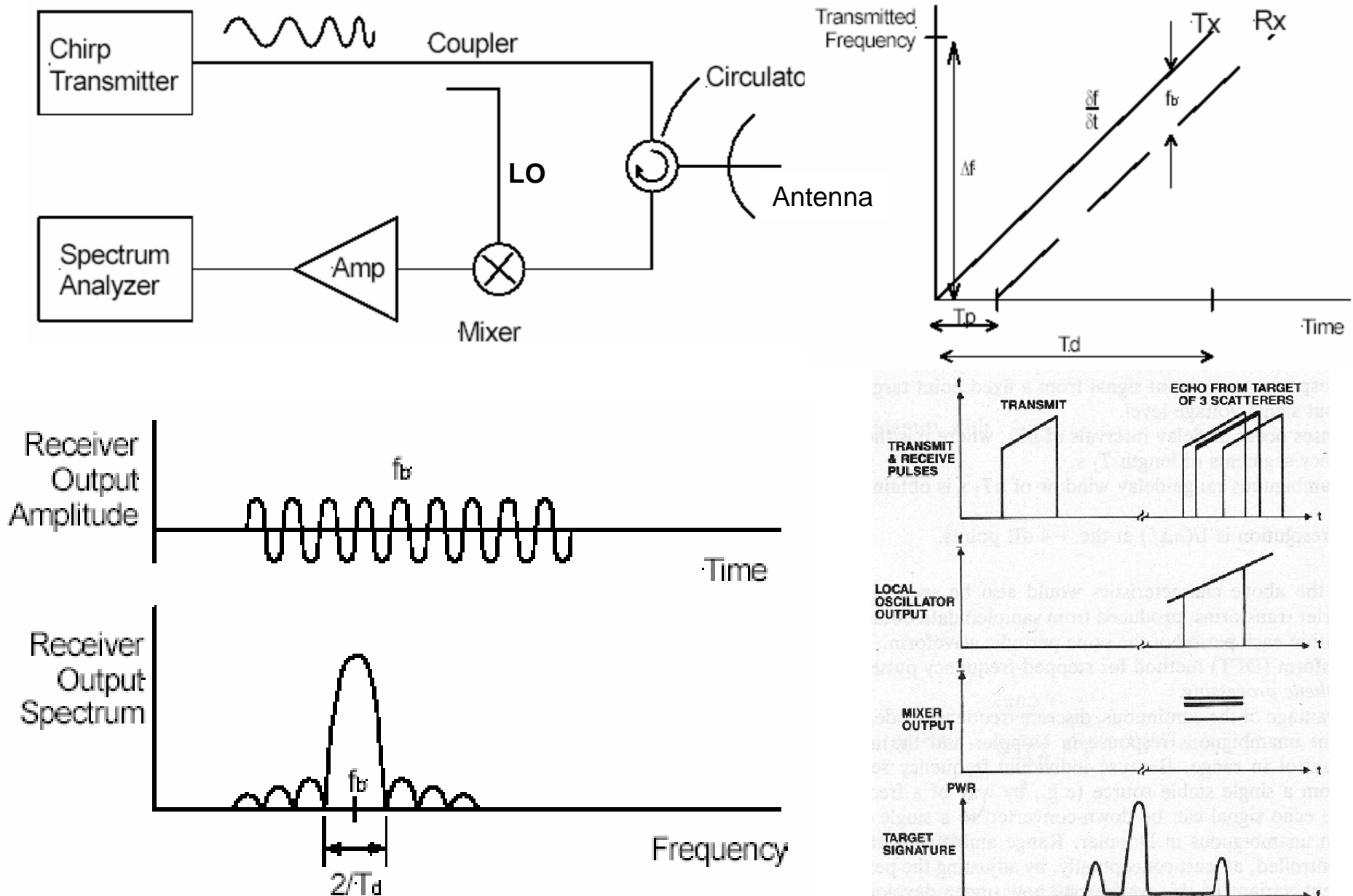
PSL: peak sidelobe level (refers to time sidelobes)

# Receiver signal processing

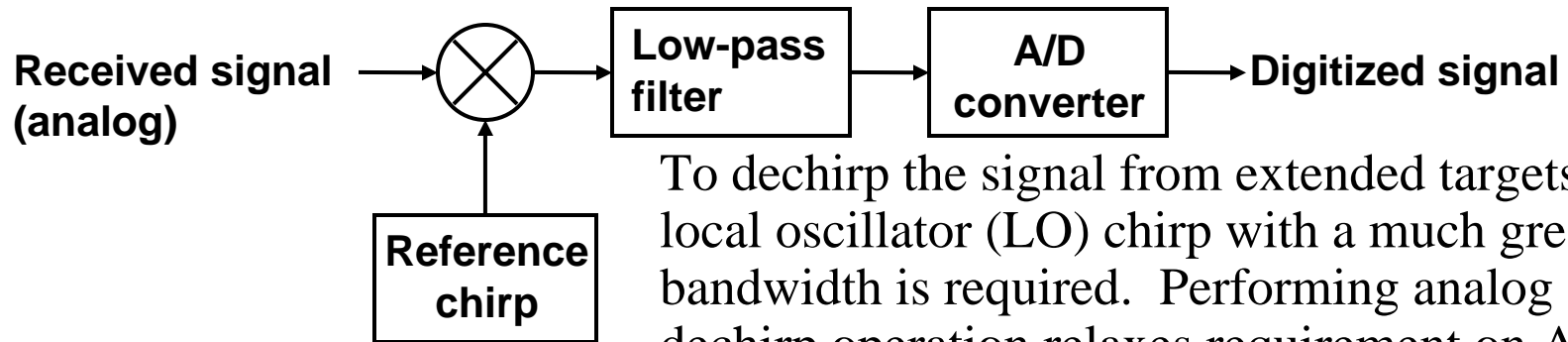
## chirp generation and compression



# Stretch chirp processing

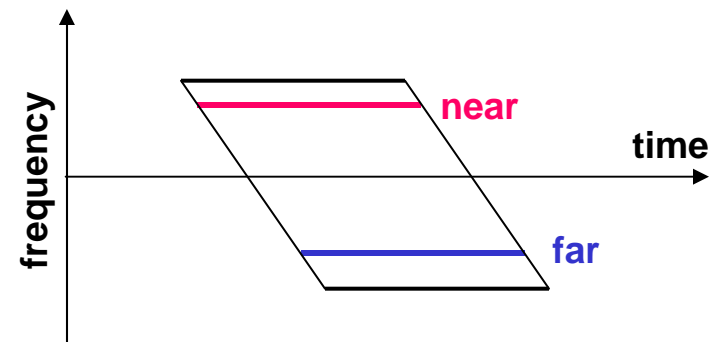
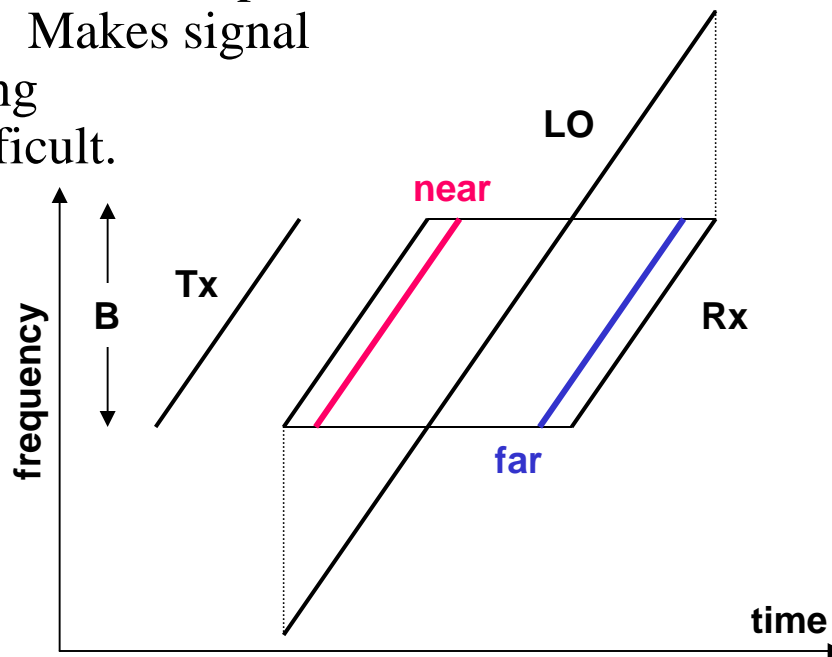


# Challenges with stretch processing



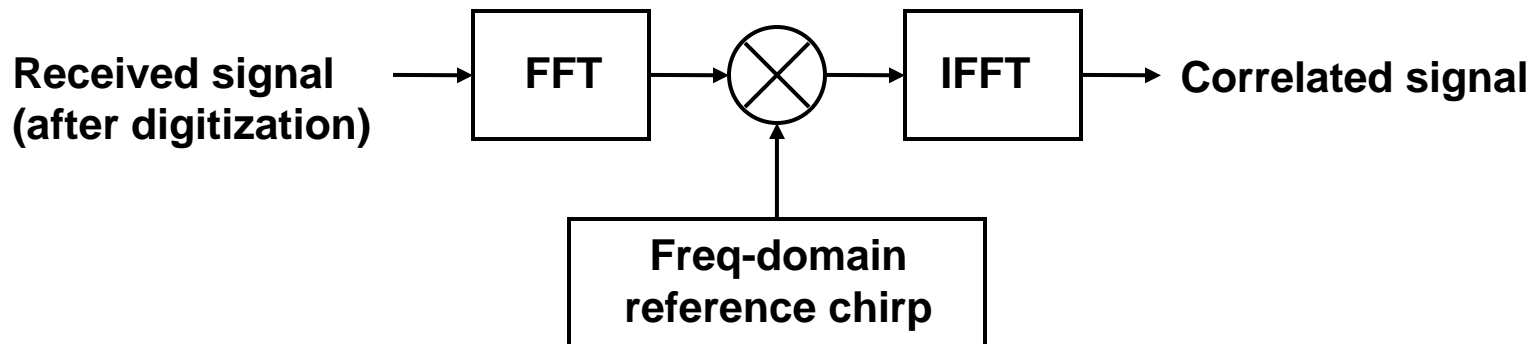
To dechirp the signal from extended targets, a local oscillator (LO) chirp with a much greater bandwidth is required. Performing analog dechirp operation relaxes requirement on A/D converter.

Echos from targets at various ranges have different start times with constant pulse duration. Makes signal processing more difficult.

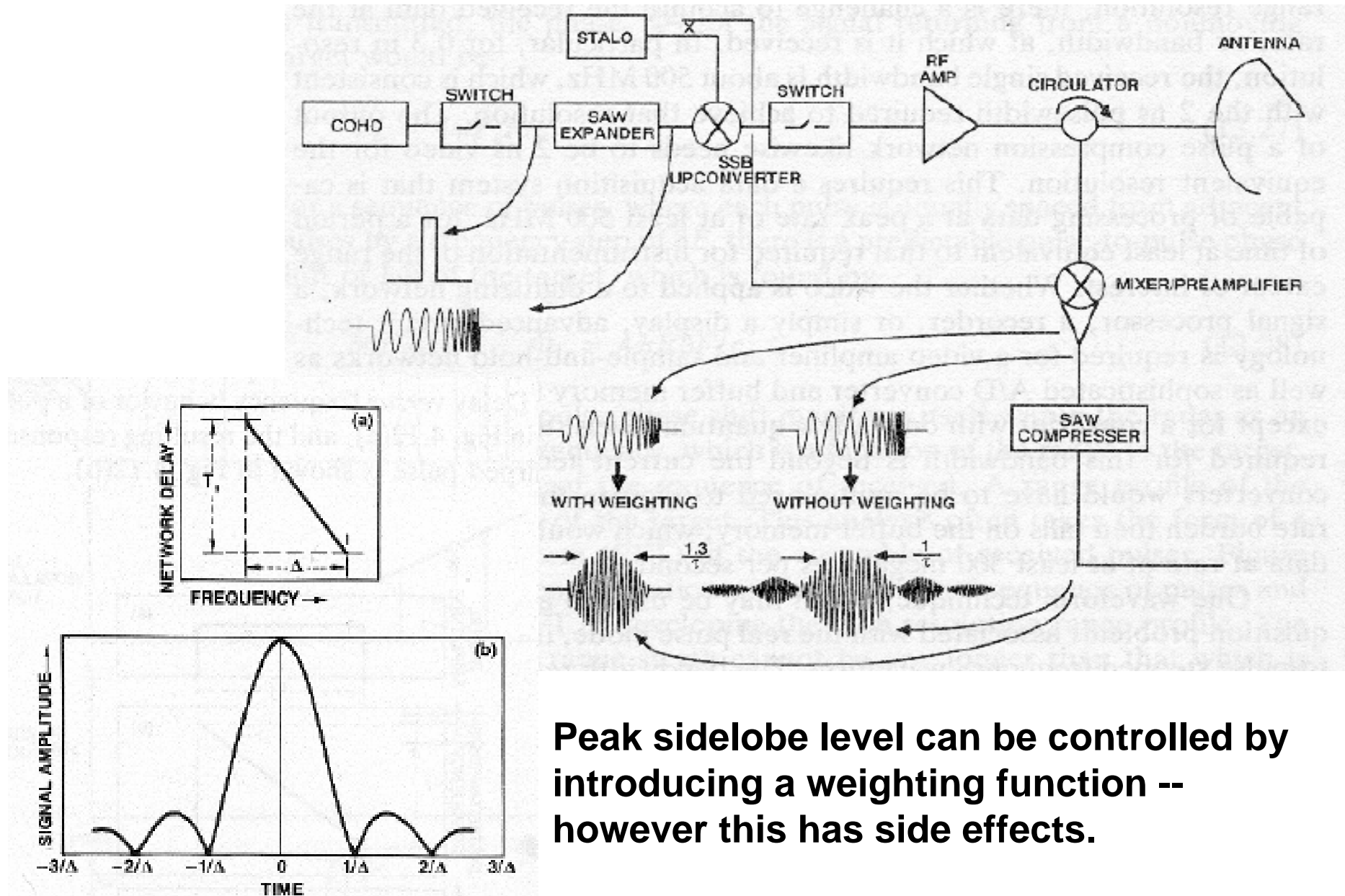


# Correlation processing of chirp signals

- Avoids problems associated with stretch processing
- Takes advantage of fact that convolution in time domain equivalent to multiplication in frequency domain
  - Convert received signal to freq domain (FFT)
  - Multiply with freq domain version of reference chirp function
  - Convert product back to time domain (IFFT)



# Chirp pulse compression and sidelobes



**Peak sidelobe level can be controlled by introducing a weighting function -- however this has side effects.**

# Window functions and their effects

Weighting Function	Peak Sidelobe Level	S/N Loss	Relative Mainlobe Width
Uniform	-13.2	0	1
$0.33+0.66\cos^2(\pi f/\beta)$	-25.7	0.55	1.23
$\cos^2(\pi f/\beta)$	-31.7	1.76	1.65
Taylor (n=8)	-40	1.14	1.41
Dolph Chebyshev	-40	-	1.35
Hamming	-42.8	1.34	1.5

**Time sidelobes are an side effect of pulse compression.**

**Windowing the signal prior to frequency analysis helps reduce the effect.**

## Some common weighting functions and key characteristics

## Less common window functions used in radar applications and their key characteristics

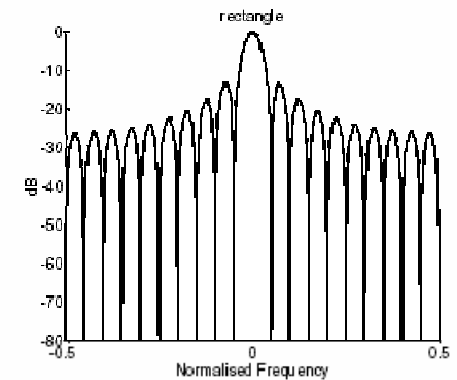
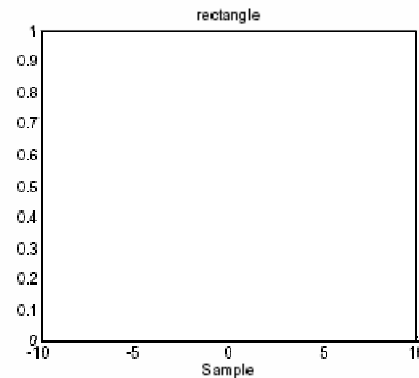
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# Window functions

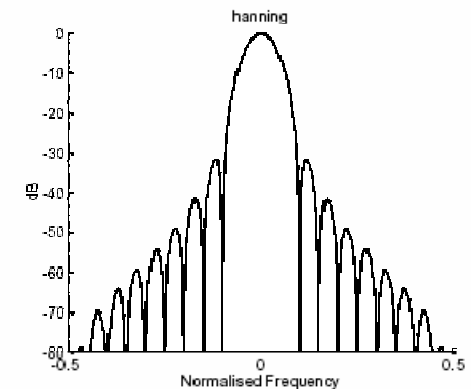
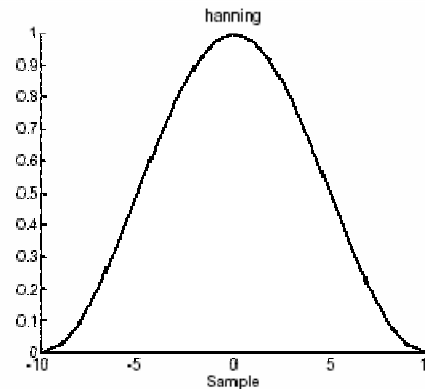
**Basic function:**  $c_k = \cos(2k\pi(n - \frac{1}{2}N) / N)$

**a and b are the -6-dB and  $-\infty$  normalized bandwidths**

Rectangular:  $w(n) \equiv 1$   
a=1.21, b=2  
Sidelobe = -13dB



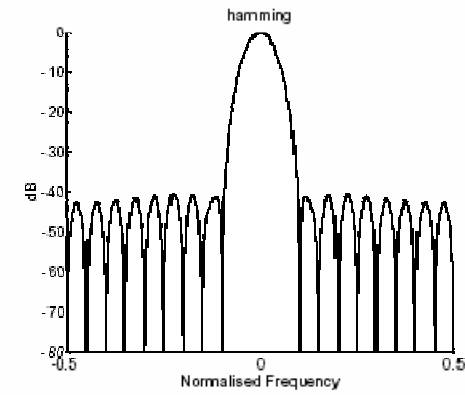
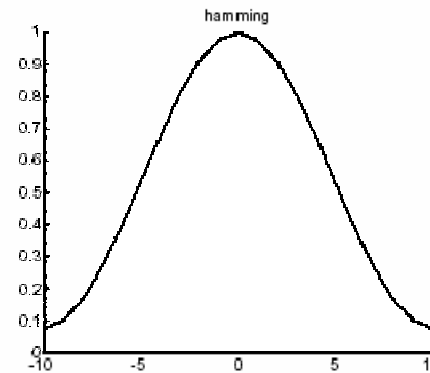
Hanning:  $0.5 + 0.5c_1$   
a=1.65, b=4  
Sidelobe = -23dB



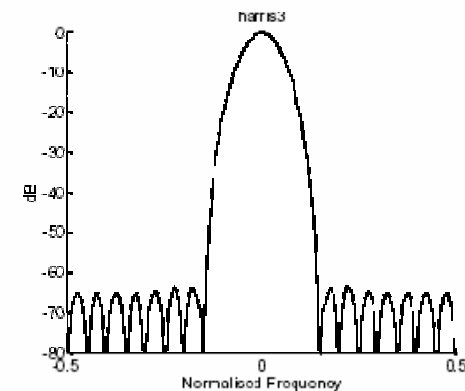
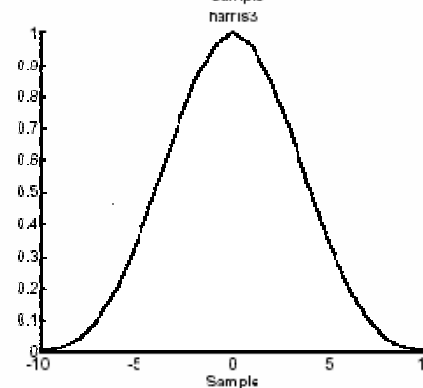


# Window functions

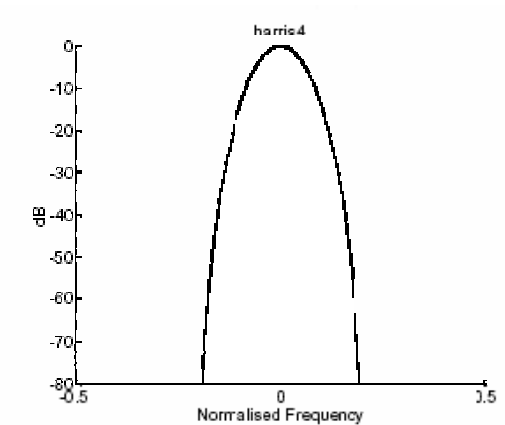
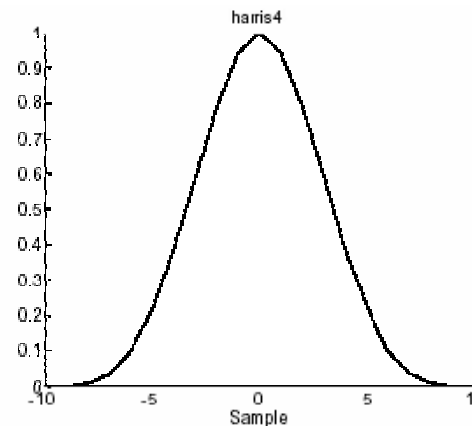
Hamming:  $0.54 + 0.46c_1$   
 $a=1.81$ ,  $b=4$   
 Sidelobe =  $-43\text{dB}$



Blackman-Harris: 3 term  
 $0.423 + 0.498c_1 + 0.079c_2$   
 $a=1.81$ ,  $b=6$   
 Sidelobe =  $-67\text{dB}$



Blackman-Harris: 4 term  
 $0.359 + 0.488c_1 + 0.141c_2 + 0.012c_3$   
 $a=2.72$ ,  $b=8$   
 Sidelobe =  $-92\text{dB}$



# Detailed example of chirp pulse compression

## received signal

$$s(t) = a \cos(2\pi(f_c t + 0.5 k t^2) + \phi_c)$$

## dechirp analysis

$$s(t) s(t - \tau) = a \cos(2\pi(f_c t + 0.5 k t^2) + \phi_c) a \cos(2\pi(f_c(t - \tau) + 0.5 k(t - \tau)^2) + \phi_c)$$

which simplifies to

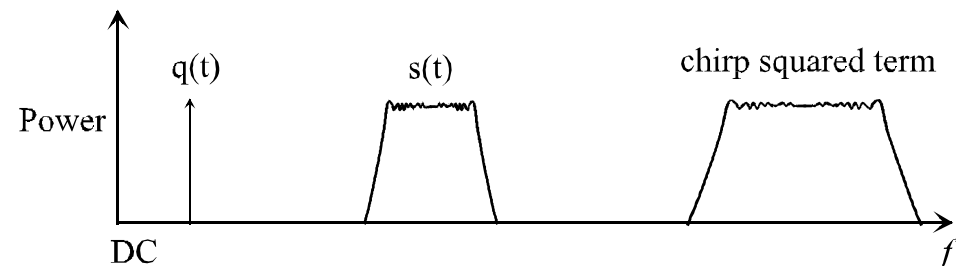
$$s(t) s(t - \tau) = \frac{a^2}{2} \left[ \cos(2\pi f_c \tau + 2\pi k t \tau - \pi k \tau^2) + \cos(2\pi(k t^2 + 2 f_c t - k \tau t + 0.5 k \tau^2 - f_c \tau) + 2\phi_c) \right]$$

Diagram illustrating the components of the simplified equation:

- sinusoidal term**: Points to  $\cos(2\pi f_c \tau + 2\pi k t \tau - \pi k \tau^2)$
- chirp-squared term**: Points to  $\cos(2\pi(k t^2 + 2 f_c t - k \tau t + 0.5 k \tau^2 - f_c \tau) + 2\phi_c)$
- quadratic frequency dependence**: Points to  $k t^2$  in the chirp-squared term.
- linear frequency dependence**: Points to  $2 f_c t$  and  $-k \tau t$  in the chirp-squared term.
- phase terms**: Points to  $2\phi_c$  in the chirp-squared term.

after lowpass filtering to reject harmonics

$$q(t) = \frac{a^2}{2} \cos(2\pi(f_c \tau + k \tau t - 0.5 k \tau^2))$$



# Pulse compression effects on SNR and blind range

SNR improvement due to pulse compression:  $B\tau$

$$\text{SNR}_{\text{compress}} = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T B F} B \tau$$

Case 1:  $P_t = 1 \text{ MW}$ ,  $\tau = 1 \text{ ns}$ ,  $B = 1 \text{ GHz}$ ,  $E = 1 \text{ mJ}$

For a given  $R$ ,  $G_t$ ,  $G_r$ ,  $\lambda$ ,  $\sigma$ :  $\text{SNR}_{\text{video}} = 10 \text{ dB}$

$B\tau = 1$  or  $0 \text{ dB}$

$\text{SNR}_{\text{compress}} = \text{SNR}_{\text{video}} = 10 \text{ dB}$

Blind range =  $c\tau/2 = \mathbf{0.15 \text{ m}}$

Case 2:  $P_t = 100 \text{ W}$ ,  $\tau = 0.1 \text{ ms}$ ,  $B = 1 \text{ GHz}$ ,  $E = 1 \text{ mJ}$

For a same  $R$ ,  $G_t$ ,  $G_r$ ,  $\lambda$ ,  $\sigma$ :  $\text{SNR}_{\text{video}} = -30 \text{ dB}$

$B\tau = 100,000$  or  $50 \text{ dB}$

$\text{SNR}_{\text{compress}} = 10 \text{ dB}$

Blind range =  $c\tau/2 = \mathbf{15 \text{ km}}$

# Conclusions

Pulse compression allows us to use a reduced transmitter power and still achieve the desired range resolution.

The costs of applying pulse compression include:

- added transmitter and receiver complexity
- must contend with time sidelobes
- increased blind range

The advantages generally outweigh the disadvantages so pulse compression is used widely.